A NOVEL FAMILY OF MATHEMATICAL SELF-CALIBRATION ADDITIONAL PARAMETERS FOR AIRBORNE CAMERA SYSTEMS

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ABSTRACT:

Self-calibration additional parameters (APs) have identified their significant role in photogrammetric calibration and orientation since 1970s. However, the traditional APs might not be adequate for the digital airborne camera calibration. A novel family of mathematical self-calibration APs is presented in this paper. We point out that photogrammetric self-calibration can, to a very large extent, be considered as a *function approximation* or, more precisely, a *curve fitting* problem in mathematics. Based on the rigorous approximation theory, the whole family of Legendre APs, which is derived from well-defined orthogonal Legendre Polynomials, is developed. Legendre APs are in general efficient to calibrate all the frame airborne cameras. They can also be considered as the superior generalization of the conventional APs developed by Ebner and Grün. A solution strategy by using Legendre APs is also suggested for in-situ camera system calibration. Extensive tests on various cameras including DMC, UltracamX, UltracamXp and DigiCAM illustrate the good performance of Legendre APs. The optimal theoretical accuracy can be achieved by applying Legendre APs, if a dense pattern of ground control points (GCPs) is available. The comparisons with the traditional APs show the theoretical and practical advantages of Legendre APs.

1. INTRODUCTION

Camera calibration is an essential subject in photogrammetry. Self-calibration by using additional parameters (APs) has been widely accepted and substantially utilized for camera calibration in photogrammetric society. Traditionally, two types of self-calibration APs were developed for analogue camera calibration: physical and mathematical. The development of physical APs was mainly attributed to D. C. Brown (Brown, 1971) for close-range camera calibration and these APs were later extended by attaching additional polynomials for aerial application (Brown, 1976). Mathematical APs (or "polynomials APs") were proposed by Ebner (1976) and Grün (1978), who used two- and four- order orthogonal bivariate polynomials respectively. The polynomials APs are often criticized as "have no foundations based on observable physical phenomena" (Clarke and Fryer, 1998), even though they can continuously reduce the image residuals. These APs, even though being widely used for many years even though in digital era, might be inadequate to fit the distinctive features of digital airborne cameras, such as push-broom, multi-head, virtual images composition, multiple image formats, etc..

A considerable progress was made recently for the digital camera calibration and some new APs have been developed. Fraser (1997) analyzed the digital close-range camera calibration, based on the classical work of Brown. Cramer (2009) and Jacobsen et al. (2010) reported comprehensive empirical tests, in which different APs were employed to compensate the image distortion. However, lots of the APs do not own solid physical or mathematical foundation and some are limited in calibration efficiency. For example, the input parameter b of Grün APs, which is $0.4\times l$ where l is the side length of the square analogue image, is set as $0.5\times(l_x+l_y)$ where l_x and l_y are the length and width of rectangular digital image format. Consequently, Grün APs are orthogonal on the generated 5×5 fictitious grid points, of which some may lay beyond the image format, as illustrated in Fig. 1. The incorporation of navigation sensors into airborne camera systems also demands calibrating the whole system rather than camera lens distortion only (Honkavaara, 2004, Cramer et al., 2010).

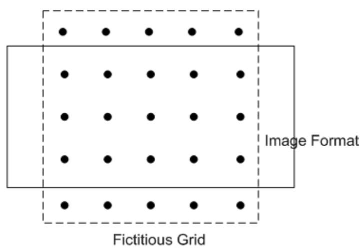

Fig. 1 the 5×5 fictitious grid points for rectangular image format

All the above motivate our present work on airborne camera calibration. We start with a mathematical viewpoint of self-calibration and then develop a novel family of polynomials APs, which has rigorous mathematical foundation. The new so-called Legendre APs are theoretically capable to calibrate the distortion of all the frame-format airborne cameras. We also suggest an easy but effective strategy for camera system calibration. Extensive empirical tests will be performed to evaluate Legendre APs in the system calibration.

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The rest is organized as follows. In Section 2, the mathematical principle of self-calibration is reviewed and Legendre APs are constructed. The practical tests are demonstrated in Section 3, followed by the comparisons between Legendre APs and traditional ones would be discussed in Section 4. This work is concluded in the final section.

2. LEGENDRE SELF-CALIBRATION APS

The collinearity equation which is the mathematical fundamental of photogrammetry reads as follows.

$$x = x_0 - c\frac{r_{11}\left(X-X_0\right)+r_{21}\left(Y-Y_0\right)+r_{31}\left(Z-Z_0\right)}{r_{13}\left(X-X_0\right)+r_{23}\left(Y-Y_0\right)+r_{33}\left(Z-Z_0\right)} + \Delta x\left(x,y\right)+\varepsilon$$
$$y = y_0 - c\frac{r_{12}\left(X-X_0\right)+r_{22}\left(Y-Y_0\right)+r_{32}\left(Z-Z_0\right)}{r_{13}\left(X-X_0\right)+r_{23}\left(Y-Y_0\right)+r_{33}\left(Z-Z_0\right)} + \Delta y\left(x,y\right)+\varepsilon \quad (1)$$

Where Δx and Δy denote image distortion, ε random error. The other notations can be seen in textbooks such like Kraus (2007). The image distortion terms, $\Delta x(x,y)$ and $\Delta y(x,y)$, are two-variable functions whose form is unknown. They have to be approximated by some models, i.e., self-calibration APs.

In general, two modeling approaches are possible. On the one hand, if the physical properties of the distortion are readily understood, then the distortion can be represented by some specific functions, like the physical APs developed by Brown. On the other hand, if the precise knowledge on the distortion is not available, we need to approximate the distortion in some more abstract means. There are advantages and disadvantages for both modeling approaches. The first model can be quite accurate but usually case-dependent, while the second one are very generally effective but may face the risk of overparameterization.

We consider the second approach. We would like to find a mathematical approach to model and compensate the distortion. It shall be orthogonal, mathematically rigorous, and generally effective for all airborne cameras.

As image measurements are available to fix the distortion, the distortion of unknown form can be approximated by the linear combination of mathematical basis functions. The coefficients can be estimated during the adjustment process. Therefore, photogrammetric self-calibration can to a very large extent be considered as a *function approximation* or, more precisely, a *curve fitting* problem in mathematics. Therefore, we will start with the general principle of the mathematical approximation as follows.

2.1 Orthogonal polynomials approximation

The algebraic polynomial approximation is founded on the Weierstrass Theorem (Mason & Handscomb, 2002). It indicates that any univariate function can be approximated with arbitrary accuracy by a polynomial of sufficiently high degree. Among all the possible forms, the orthogonal polynomials (OPs) are often favored in both theoretical and practical applications due to many elegant properties. The OPs can be categorized into two types: discrete and continuous. The former is orthogonal on specific discrete measurements while the later is orthogonal over the whole domain of definition.

For the curve fitting problem, the analytical form of the function is unknown while some sample measurements are available. The unknown function can be approximated by the combination of OPs. If the number of the measurements is close to the degree of the used polynomials, the discrete OPs are usually employed and can be obtained by orthogonalization process. It is noteworthy that the discrete OPs are orthogonal on the measured locations only, but not necessarily on the others. Else, if the measurements are very dense and the number is much larger than the polynomials' degree, the continuous OPs is preferred. More theoretical materials can be seen in such as Berztiss (1964) and Mason & Handscomb (2002).

Legendre Polynomials, denoted by $\{L_m(x)\}_{m=0,1,...}$ where m indicates the order, are a series of continuous OPs over $[-1,1]$, i.e.,

$$\left|L_m(x)\right| \le 1, \quad -1 \le x \le 1 \quad (2)$$

$$\int_{-1}^{1} L_m(x) L_n(x) dx = \begin{cases} 0, & m \ne n \\ 1, & m = n \end{cases} \quad (3)$$

Legendre polynomials grant the optimal approximation in the least-square sense (Mason & Handscomb, 2002) and are widely used in many applications. The first few normalized Legendre Polynomials are listed in appendix A.

The bivariate OPs can be generalized from the univariate cases. They could be much more complicated, depending on the two-dimensional definition domain. Particularly, the two-dimensional generalization on the rectangular domain turns out to be rather straightforward. Namely, if $\{p_m(x)\}_{m=0,1,...}$ are a series of univariate OPs over $[-1,1]$, then

$$\left\{ p_{m,n}(x,y) = p_m(x) p_n(y) \right\}_{m=0,1,...\ \ n=0,1,...} \quad (4)$$

are complete bivariate OPs over the rectangular domain $[-1,1]\times[-1,1]$, satisfying

$$\int_{-1}^{1}\int_{-1}^{1} p_{m,n}(x,y) p_{i,j}(x,y) dxdy = \begin{cases} 1, & \text{if } m=i \text{ and } n=j \\ 0, & \text{else} \end{cases} \quad (5)$$

"Complete" indicates that any two-variable function can be approximated well by the $\left\{p_{m,n}\right\}$ (Koornwinder, 1975).

2.2 Self-calibration APs

Let $2b_x$ and $2b_y$ denote the width and length of the image format, respectively. By scaling we obtain,

$$l_m(x,b_x) = L_m\left(x/b_x\right) \quad (6)$$
$$l_n(y,b_y) = L_n\left(y/b_y\right) \quad (7)$$

where x and y are the metric image coordinates, L_m and L_n are univariate Legendre Polynomials $(m,n=0,1,2,...)$. The first few $\left\{l_m(x,b_x)\right\}_m$ are,

$$l_0(x,b_x) = 1$$
$$l_1(x,b_x) = x/b_x$$

$$l_2(x,b_x)=\left(3\left(x/b_x\right)^2-1\right)/2$$
$$l_3(x,b_x)=\left(5\left(x/b_x\right)^3-3\left(x/b_x\right)\right)/2$$
$$l_4(x,b_x)=\left(35\left(x/b_x\right)^4-30\left(x/b_x\right)^2+3\right)/8$$
$$l_5(x,b_x)=\left(63\left(x/b_x\right)^5-70\left(x/b_x\right)^3+15\left(x/b_x\right)\right)/8$$
$$l_6(x,b_x)=\left(231\left(x/b_x\right)^6-315\left(x/b_x\right)^4 +105\left(x/b_x\right)^2-5\right)/16$$

Similar formulae of $\left\{l_n(y,b_y)\right\}_n$ could be derived. Denote

$$f_{m,n}\triangleq f_{m,n}(x,y;b_x,b_y)=l_m(x,b_x)l_n(y,b_y) \quad (8)$$

then $\left\{f_{m,n}\right\}_{m,n}$ are the bivariate OPs over the rectangular frame $\left[-b_x,b_x\right]\times\left[-b_y,b_y\right]$ and $\left|f_{m,n}\right|\le 1$. Considering the image distortion is typically in the order of μm, we obtain $p_{m,n}$ by multiplying $f_{m,n}$ with 10^{-6} for numerical stability.

$$p_{m,n}=10^{-6}f_{m,n},\ \left|p_{m,n}\right|\le 10^{-6} \quad (9)$$

$\left\{p_{m,n}\right\}_{m,n}$ can be ordered lexicographically as Eq. (10), following Koornwinder (1975).

$$\begin{array}{l} p_{0,0} \\ p_{1,0},\ p_{0,1} \\ p_{2,0},\ p_{1,1},\ p_{0,2} \\ p_{3,0},\ p_{2,1},\ p_{1,2},\ p_{0,3} \\ p_{4,0},\ p_{3,1},\ p_{2,2},\ p_{3,1},\ p_{0,4} \\ \ldots\ldots \end{array} \quad (10)$$

Obviously,

$$\int_{-b_x}^{b_x}\int_{-b_y}^{b_y} p_{i,j}p_{m,n}dxdy=0 \quad \text{if} \quad i\ne m \quad \text{or} \quad j\ne n \quad (11)$$

It indicates that if the image measurements are densely distributed, then

$$\sum_k p_{i,j}(x_k,y_k)p_{m,n}(x_k,y_k)\approx 0 \quad \text{if} \quad i\ne m \quad \text{or} \quad j\ne n \quad (12)$$

Eq. (12) implies that $\left\{p_{m,n}\right\}_{m,n}$ is (almost) orthogonal over the all image measurements.

Therefore, the bivariate distortion $\Delta x(x,y)$ ($\Delta y(x,y)$) in Eq. (1) could be approximated by a series of continuous OPs $\left\{p_{m,n}\right\}_{m=0,n=0}^{m=M_x,n=N_x}$ ($\left\{p_{m,n}\right\}_{m=0,n=0}^{m=M_y,n=N_y}$), where M_x and N_x (M_y and N_y) are the chosen maximum degrees which are not necessarily equal. Further, six of them should be eliminated, as done by Ebner (1976) and Grün (1978). Specially, the constant terms $p_{0,0}$ in $\Delta x(x,y)$ and $\Delta y(x,y)$ are nothing but the principle point offset; $p_{1,0}$, $p_{0,1}$, $p_{2,0}$ and $p_{1,1}$ in $\Delta x(x,y)$ are highly correlated with $p_{0,1}$, $p_{1,0}$, $p_{1,1}$ and $p_{0,2}$ in $\Delta y(x,y)$, respectively. Thus, the number of the unknown parameters is $(M_x+1)(N_x+1)+(M_y+1)(N_y+1)-6$.

As examples, the APs with $M_x=M_y=5$ and $N_x=N_y=5$ are obtained in Eq. (13) with 66 unknown parameters (a_i, $i=1,2,...,66$). The APs with 34 unknowns, $M_x=M_y=4$ and $N_x=N_y=3$, are given in Eq. (14).

So far the whole family of APs has been completely constructed. The input of APs includes the image length and width ($2b_x$ and $2b_y$), and the chosen degrees (M_x, N_x, M_y and N_y). Usually, it can further adopt $M_x=M_y\triangleq M$ and $N_x=N_y\triangleq N$ in practice. This class of APs is based on Legendre Polynomials and thus called Legendre APs.

$$\begin{aligned}
\Delta x &= a_1p_{1,0}+a_2p_{0,1}+a_3p_{2,0}+a_4p_{1,1}+a_5p_{0,2}+a_6p_{3,0}+a_7p_{2,1}\\
&\quad +a_8p_{1,2}+a_9p_{0,3}+a_{10}p_{4,0}+a_{11}p_{3,1}+a_{12}p_{2,2}+a_{13}p_{1,3}\\
&\quad +a_{14}p_{0,4}+a_{15}p_{5,0}+a_{16}p_{4,1}+a_{17}p_{3,2}+a_{18}p_{2,3}+a_{19}p_{1,4}\\
&\quad +a_{20}p_{0,5}+a_{21}p_{5,1}+a_{22}p_{4,2}+a_{23}p_{3,3}+a_{24}p_{2,4}+a_{25}p_{1,5}\\
&\quad +a_{26}p_{5,2}+a_{27}p_{4,3}+a_{28}p_{3,4}+a_{29}p_{2,5}+a_{30}p_{5,3}\\
&\quad +a_{31}p_{4,4}+a_{32}p_{3,5}+a_{33}p_{5,4}+a_{34}p_{4,5}+a_{35}p_{5,5}\\
\Delta y &= a_2p_{1,0}-a_1p_{0,1}+a_{36}p_{2,0}-a_3p_{1,1}-a_4p_{0,2}+a_{37}p_{3,0}+a_{38}p_{2,1}\\
&\quad +a_{39}p_{1,2}+a_{40}p_{0,3}+a_{41}p_{4,0}+a_{42}p_{3,1}+a_{43}p_{2,2}+a_{44}p_{1,3}\\
&\quad +a_{45}p_{0,4}+a_{46}p_{5,0}+a_{47}p_{4,1}+a_{48}p_{3,2}+a_{49}p_{2,3}+a_{50}p_{1,4}\\
&\quad +a_{51}p_{0,5}+a_{52}p_{5,1}+a_{53}p_{4,2}+a_{54}p_{3,3}+a_{55}p_{2,4}+a_{56}p_{1,5}\\
&\quad +a_{57}p_{5,2}+a_{58}p_{4,3}+a_{59}p_{3,4}+a_{60}p_{2,5}+a_{61}p_{5,3}\\
&\quad +a_{62}p_{4,4}+a_{63}p_{3,5}+a_{64}p_{5,4}+a_{65}p_{4,5}+a_{66}p_{5,5}
\end{aligned} \quad (13)$$

$$\begin{aligned}
\Delta x &= a_1p_{1,0}+a_2p_{0,1}+a_3p_{2,0}+a_4p_{1,1}+a_5p_{0,2}+a_6p_{3,0}+a_7p_{2,1}\\
&\quad +a_8p_{1,2}+a_9p_{0,3}+a_{10}p_{4,0}+a_{11}p_{3,1}+a_{12}p_{2,2}+a_{13}p_{1,3}\\
&\quad +a_{14}p_{4,1}+a_{15}p_{3,2}+a_{16}p_{2,3}+a_{17}p_{4,2}+a_{18}p_{3,3}+a_{19}p_{4,3}\\
\Delta y &= a_2p_{1,0}-a_1p_{0,1}+a_{20}p_{2,0}-a_3p_{1,1}-a_4p_{0,2}+a_{21}p_{3,0}+a_{22}p_{2,1}\\
&\quad +a_{23}p_{1,2}+a_{24}p_{0,3}+a_{25}p_{4,0}+a_{26}p_{3,1}+a_{27}p_{2,2}+a_{28}p_{1,3}\\
&\quad +a_{29}p_{4,1}+a_{30}p_{3,2}+a_{31}p_{2,3}+a_{32}p_{4,2}+a_{33}p_{3,3}+a_{34}p_{4,3}
\end{aligned} \quad (14)$$

2.3 Overall system calibration

Nowadays the integrated navigation systems are incorporated as a part of the digital airborne camera systems. The GPS/IMU incorporation, on the one hand, accelerates the photogrammetric mapping and remarkably reduces the number of ground control points (GCPs). On the other hand, it brings extra systematic effects, e.g., the misalignment between the camera and the navigation instruments and the drift/shift effect in the direct georeferencing measurements. Therefore, the overall camera system calibration becomes a must in current photogrammetry. For the system calibration, one most challenging work could be to minimize the coupling effect of the different systematic errors. The decoupling is of vital importance in the sense that each systematic error must be independently and appropriately calibrated and the calibration results are block-invariant.
For this purpose, we suggest the joint application of the Legendre APs (for calibrating the image distortion) with other correction parameters, i.e., the three interior orientation (IO) parameters used for correcting the principle point offset and the focal length deformation, and GPS/IMU drift/shift and misalignment correction parameters. The low correlation must be warranted among these calibration parameters and between them and exterior orientation (EO). As will be seen in Section 4, the correlations between Legendre APs and EO, and between Legendre APs and other correlation parameters, are fairly small. The low correlation is one advantage of Legendre APs over the traditional APs.

3. PRACTICAL TESTS

The Legendre APs are empirically tested by using the data from the recent DGPF project (German Society for Photogrammetry, Remote Sensing and Geoinformation), which was performed under the umbrella of DGPF and carried out in the test field Vaihingen/Enz nearby Stuttgart, Germany. This project aims at an independent and comprehensive evaluation on the performance of digital airborne cameras, as well as offering a standard empirical dataset for the next years.
Four flights' data of the frame cameras are adopted: DMC (GSD 20cm, ground sample distance), DMC (GSD 8cm), UltracamX (GSD 20cm) and UltracamX (GSD 8cm). For each flight, we are interested in two most often contexts: the in-situ calibration one and the operational project one. The former context is with high side overlapping (≈60%) and dense GCPs and the later with low side overlapping (≈20%) and few GCPs. The block configuration of four flights is not detailed here and the readers are referred to Cramer (2010) and DGPF website (2010) for the project details as well.

3.1 In-situ calibration context

The system calibration strategy in Section 2.3 is adopted for all the blocks. Particularly, IMU misalignment, horizontal GPS shift, IO parameters and Legendre APs with $M_x = N_x = M_y = N_y = 5$ are employed. The order of Legendre APs is empirically selected by the compromise between achieving the optimal accuracy and reducing overparameterization. The derived external accuracy, indicated by "self calibrating", would be compared to the theoretical accuracy and the "without APs" one, for which the same correction parameters except Legendre APs are used.
The derived external accuracy is demonstrated in Fig. 2. By comparing "Self calibrating" with "Without APs", the refinement of Legendre APs is significant in all tests, up to 10 cm in the DMC (GSD 20cm) block. Moreover, all the "self calibrating" accuracy reaches very close to the theoretical one and it means that the optimal accuracy has been achieved. The closeness keeps well when the presumptions of the std. dev. of the GPS observations are varied from 2cm to 20cm (though not illustrated here). All the "self calibrating" accuracy reaches to 1/5 GSD in the horizontal directions and 2/5 GSD in the vertical directions in four blocks. It is also interesting to notice that although the DMC and UltracamX cameras are differently manufactured, very similar external accuracy can be obtained by using Legendre APs in the blocks of similar configuration, i.e., similar GSD, similar forward and side overlapping levels and similar GCPs distribution. This fact, independent of the used cameras, coincides well with our photogrammetric accuracy expectation.
Now look at the estimation of the precision of the image measurements. The posterior std. dev. estimation is 1.6, 1.4, 0.89 and 0.78 (unit: μm) for DMC (GSD 20cm, GSD 8cm) and UltracamX (GSD 20cm, GSD 8cm) blocks, respectively. These values are around 0.12 pixel size, which are 12 μm and 7.2 μm for DMC and UltracamX cameras, respectively. They well match the expected precision of the automatic tie point transfer techniques, which are 0.1-0.2 pixel size for aerial images.

3.2 Operational project context

There are 4 GCPs and 20% side overlapping level in each block, which is much weaker than the in-situ calirbation context. The IMU misalignment, IO parameters and the Legendre APs with $M_x = M_y = 4, N_x = N_y = 3$ are employed in the adjustment. Using Legendre APs of lower degree tries to avoid the potential overparameterization. This derived external accuracy is analogously denoted as "self calibrating" one. Due to 4 GCPs available only, the GPS/IMU observations have to be weighted carefully to achieve best accuracy.
We also evaluate the quality of the in-situ calibration in last sub-section. The calibration results of IO and image distortion in Section 3.1 are utilized as known and fixed values in the adjustment of the corresponding "reduced" operational block, i.e., the cameras are assumed being calibrated and need no further self-calibration. The derived external accuracy is named as "after calibration". We compare "after calibration" with "self calibrating", "without APs" and theoretical ones.
The adjust accuracy in four blocks is illustrated in Fig. 3. From those results, the self-calibrating Legendre APs help improve the external accuracy and the "after calibration" yields further refinement, more than 1/2 GSD in DMC (GSD 8cm) block. The "after calibration" accuracy is very close to the optimal theoretical one in every block. Therefore, these tests not only recognize the sufficient accuracy obtained by Legendre APs in the operational projects, but also confirm again their great efficiency in the in-situ calibration. More discussions would be appeared in Tang et. al (2012).

It is worth mentioning that the Legendre APs have also been assessed by the flight data of other airborne cameras in other test fields, like medium-format DigiCAM and large-format UltracamXp. The similar good results are confirmed while the details are not published here.

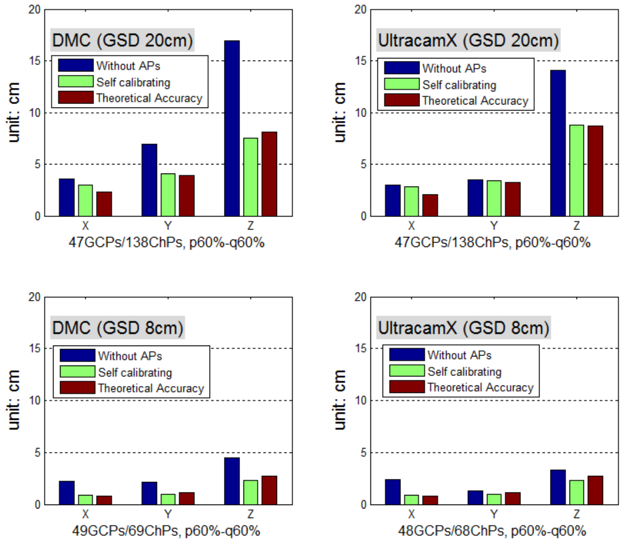

Fig. 2 External accuracy in four in-situ calibration blocks, dense GCPs and p60%-q60% ('without APs' indicates without using Legendre APs only)

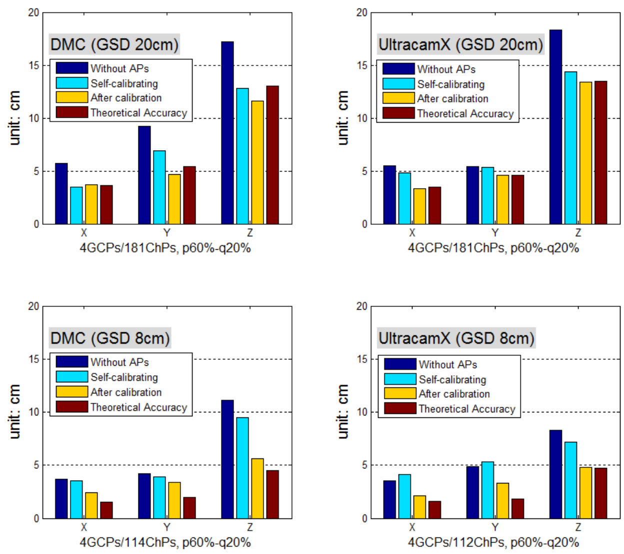

Fig. 3 External accuracy in four operational project blocks, 4 GCPs and p60%-q20% ('without APs' indicates without using Legendre APs only)

4. DISCUSSIONS

In this section, we make comparisons between Legendre APs and the conventional APs from the theoretical and practical viewpoints.

Based on the "standard" 60% forward overlapping level and a small few photographic measurements in analogue time, Ebner and Grün built polynomials APs of two and four orders, respectively. These APs are restricted in the assumed regular 3×3 and 5×5 "grid points" configurations, respectively. Both APs can be obtained by orthogonalizaiton and elimination of six highly correlated parameters, and finally get the models of 12 and 44 unknown parameters respectively. It is known, somewhat confusingly, that both APs can improve the accuracy even when the regular grid patterns are not satisfied. This is a source of criticism posed on these polynomials APs.

All these bewilderments can be clarified easily by using the theory of function approximation. First, Ebner and Grün APs are merely special orthogonal arrangements of polynomials. In fact, they belong to discrete OPs in the mathematical jargon; see the mathematical materials in Section 2.1. Second, the mathematical principle behind Ebner and Grün APs is still Weierstrass theorem, exactly the same with Legendre APs. Therefore, the irregular patterns only affect the correlations among APs rather than their effect in compensating lens distortion. That is why they still work even though tie points do not satisfy the grid pattern. Third, it is also easy to understand from the approximation view why Ebner APs sometimes achieve quite poor performance. That is, the distortion is too complex for two-order polynomials to well approximate. Higher degree's polynomials are required and that is the reason why Grün APs perform better in general. Two examples are illustrated for the comparison on the external accuracy in DMC (GSD 20cm) calibration and operational blocks, in Fig. 4 and Fig. 5 respectively. It is clear that Ebner APs obtain quite poor accuracy, particularly in Fig. 4.

However, Legendre APs must be preferred to these two traditional polynomials APs. First, from the mathematical viewpoint, continuous polynomials are more reasonable than discrete polynomials for calibration purpose, i.e., Legendre APs are more theoretically reasonable than these two APs. This is basically due to the wide-accepted assumption that the geometric distortion of each image is homogeneous in a single photogrammetric block. It implies equivalently that all measurements in all images are put into one single image dimension for the calibration purpose. Consequently, very dense image measurements encompass the image format for almost all the cases. As a simple example, considering a block including 50 images, each image contains a very small amount of measurements, say around 30. It turns out to be roughly 1500 measurements usable for self-calibration, much more than the number of unknown APs (usually quite smaller than 100). Therefore the continuous polynomials (Legendre polynomials) should be favored. Second, Ebner and Grün APs are single order polynomials while Legendre APs are a whole family of polynomials. Thus, Legendre APs offer much more flexibility for applications. Third, Legendre APs are advantageous in low correlations. An example in DMC (GSD 20cm) calibration block is illustrated in Table. 1, where '<0.1' indicates the percentage of correlations smaller than 0.1 and 'max' denotes the maximum correlations. It is demonstrated that Legendre APs have much lower correlations with IO and IMU than Grün APs. The intra-correlations among APs (denoted by 'intra-corr') show that Legendre APs are 'more orthogonal'. Fig. 4 and Fig. 5 also illustrate that Legendre APs deliver slightly better accuracy. In fact, Legendre APs can also be seen as the superior generalization of the traditional polynomials APs from the mathematical viewpoint.

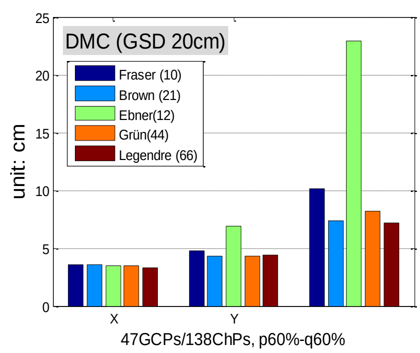

Fig. 4 Comparison on the external accuracy in DMC (GSD 20cm) calibration block (47GCPs/138ChPs, p60%-q60%)

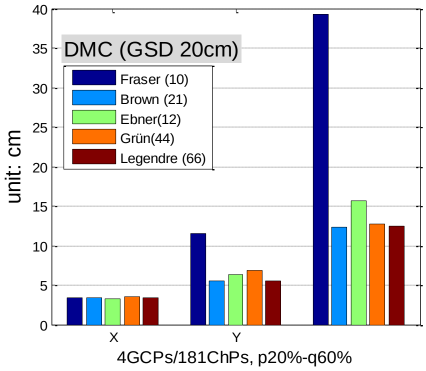

Fig.5 Comparison on the external accuracy in DMC (GSD 20cm) operational block (4GCPs/181ChPs, p20%-q60%)

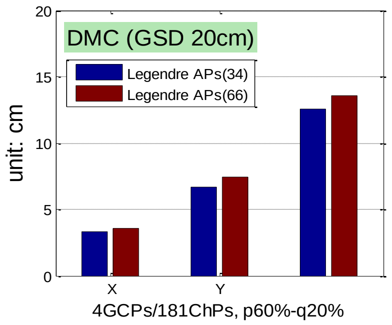

Fig. 6 Insignificant impact of moderate overparameterization on the external accuracy in DMC (GSD 20cm) operational block (4GCPs/181ChPs, p20%-q60%)

Table 1. Correlation analysis in DMC (GSD 20cm) calibration block (47GCPs/138ChPs, p60%-q60%)

APs	corr.	EO	IO	IMU	Intra-corr
Grün APs (44)	< 0.1	100%	80%	83%	88%
	max	---	0.73	0.53	0.93
Brown APs (21)	< 0.1	98%	78%	86%	78%
	max	0.19	0.87	0.55	0.92
Legendre APs (66)	< 0.1	100%	97%	100%	96%
	max	---	0.44	---	0.57

We also compare Legendre APs with the physical APs proposed in Brown (1971) (Fraser (1997) as well) and Brown (1976). Although Brown model achieves the comparable accuracy as Legendre APs do in Fig. 4 and Fig. 5, the later possesses much better performance in low correlations, as demonstrated in Table 1. In fact, the image distortion of the multi-head airborne cameras is not dominated by the radial-symmetric distortion anymore; and this is the main reason why Fraser model delivers rather poor accuracy (Fig. 5).

The mathematical APs are sometimes criticized for overparameterization. It is true that mathematical APs achieve general effectiveness at the price of more parameters than physical counterparts. However, we urge here that moderate overparameterization is tolerable and would not degrade remarkably the accuracy. Fig. 6 depicts an example of DMC (GSD 20cm) operational block (2 flight lines, 4 GCPs, 20% side overlap and 28 images). The Legendre APs of 66 and 34 unknowns are applied. It is seen that no significant difference is observed. The influence of redundant APs can be further reduced in the blocks of higher overlapping and more GCPs. From the view of curve fitting, this tolerance of moderate redundant APs is mainly due to the number of image measurements being much larger than number of APs.

5. CONCLUSIONS

We proposed a new class of self-calibration Legendre APs for calibrating digital frame-format airborne cameras. The prime theoretical foundations of Legendre APs are mathematical polynomial approximation and the renowned Weierstrass Theorem. This is one significant theoretical development in this work. It is thus guaranteed that Legendre APs of proper degree can, at least theoretically, calibrate the distortion of all the frame cameras.

The excellent performance of Legendre APs is demonstrated in the extensive tests on various airborne cameras, including DMC, DigiCam, UltracamX and UltracamXp. Legendre APs are generally effective and flexible for calibrating all digital frame airborne camera architectures, no matter which system design have been chosen by the camera manufacturer. In principle, they can be used for calibrating frame cameras of large-, medium- and small-format CCDs, mounted in single- and multi-head systems. Moreover, the very low correlation between Legendre APs and other parameters, such as those for exterior orientation (EO) and GPS/IMU offsets or misalignments, guarantees reliable calibration results.

We also compare Legendre APs with other traditional APs. Both the theoretical investigations and practical experiments show Legendre APs are superior to the conventional Ebner and Grün APs. Compared with the physical APs, Legendre APs show its advantages in general efficiency and very low correlations.

Although the polynomials APs are sometimes dubbed 'empirical' (McGlone et al., 2004), Legendre APs are in fact 'more objective' in many senses. It is allowed to conclude that Legendre APs are orthogonal, rigorous, generic and effective for the digital frame airborne cameras' calibration.

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APPENDIX

The first few normalized orthogonal Legendre Polynomials over the interval $\left[-1,1\right]$ are following.

$$L_0(x) = 1$$

$$L_1(x) = x$$

$$L_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$L_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$L_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$L_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

$$L_6(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$$