A MATHEMATICAL MODEL FOR CAMERA CALIBRATION USING STRAIGHT LINES

A. M. G. Tommaselli [a, *], S. S. S. Telles [b]

[a] Department of Cartography, FCT-Unesp, 19060-900 Pres. Prudente, SP, Brazil – tomaseli@prudente.unesp.br
[b] Atech – Fundação Tecnologias Críticas - stelles_sol@hotmail.com

Commission I, WG I/3

KEY WORDS: Metrology**,** Calibration, Straight Line, Non-Metric, Digital.

ABSTRACT:

The aim of this paper is to present the derivation and experimental evaluation of a simultaneous method for camera calibration using straight lines. A mathematical model using straight lines was derived to consider both interior and exterior orientation parameters as unknowns. The interior orientation parameters to be estimated are the camera focal length, the coordinates of the principal point and lens distortion parameters (radial and decentering). The mathematical model is based on the equivalence between the vector normal to the interpretation plane in the image space and the vector normal to the rotated interpretation plane in the object space. This model was implemented and tests with simulated and real data were performed. The discussion of the obtained results are presented in this paper. The results with the proposed model were compared with those obtained with conventional self calibrating bundle adjustment and it is shown that the results are similar.

1. INTRODUCTION

Camera calibration is a fundamental task in Photogrammetry and Computer Vision. The lack of accurate inner orientation parameters leads to unreliable results in the photogrammetric process. Calibration and orientation of digital images using lines have gained interest, basically due to the potential of automation and the robustness of some methods of line detection. Another remarkable advantage of using lines is the number of observations that can be provided, improving significantly the overall system redundancy.

The classical calibration or space resection methods solve the problem by using points in a bundle adjustment. The non-linearity of the model and problems in point location in digital images are the main drawbacks of these approaches. The proposed solutions to overcome these problems include either the use of linear models (Lenz and Tsai, 1988) or the use of linear features: Lugnani (1980), Masry (1981), Tommaselli and Lugnani (1988), Mulawa and Mikhail (1988), Haralick (1989), Liu et al (1990), Wang and Tsai (1990), Lee et al (1990), Tommaselli and Tozzi (1996), Quan and Kanade (1997), Habib et al (2002), Habib et al (2003), Schenk (2004). Most of these works deals only with Exterior Orientation Parameters (EOP). Ethrog (1984), however, presented a photogrammetric method for determining the Interior Orientation Parameters (IOP) and the orientation angles, using objects with parallel and perpendicular lines, considering photographs taken with non-metric cameras.

One of the earliest approaches using lines in Photogrammetry was the plumb line calibration method (Brown, 1971). This method is suitable to recover the radial and decentering lens distortion coefficients, while the remaining interior (focal length and principal point coordinates) and exterior orientation parameters have to be determined by a complimentary method. A similar method was presented by Prescott and McLean (1997) who compared their line based method for calibration of radial lens distortion with the point based linear method of Tsai (Lenz and Tsai, 1988) and reported similar results showing the accuracy potential of line based approaches. Tommaselli (2000) and Telles and Tommaselli (2002) proposed and additional step in which the coordinates of the principal point and the focal length could be computed using lines.

Several other available methods using lines, consider the determination of exterior orientation parameters, with no mention to the simultaneous determination of inner orientation parameters, which are considered known from previous calibration.

The digital cameras produced for the consumer market differ in size, cost and stability of the inner geometry in comparison to the metric analog cameras. As a consequence, when using digital non-metric cameras for metric purposes, it is recommended to use on-the-job calibration methods, or rely on periodic calibrations. Calibration fields with straight lines are easy to build and this will facilitate periodic calibrations of digital cameras.

The main goal of this work was to derive a mathematical model relating image and object space using straight lines and considering also the interior orientation parameters as unknowns. This model was implemented and tested with simulated and real data. The results obtained with the proposed model were compared with those obtained with conventional calibration with bundle adjustment.

2. MATHEMATICAL MODEL

In this section the mathematical model using straight lines will derived considering both interior and exterior orientation parameters as unknowns. The interior orientation parameters to be estimated are the camera focal length, the coordinates of the principal point and lens distortion parameters (radial and decentering). The mathematical model is based on the equivalence between the vector normal to the interpretation plane in the image space and the vector normal to the rotated interpretation plane in the object space, and it is an expansion of the equivalent plane model, proposed by Tommaselli and Lugnani (1988) and revived by Tommaselli and Tozzi (1996).

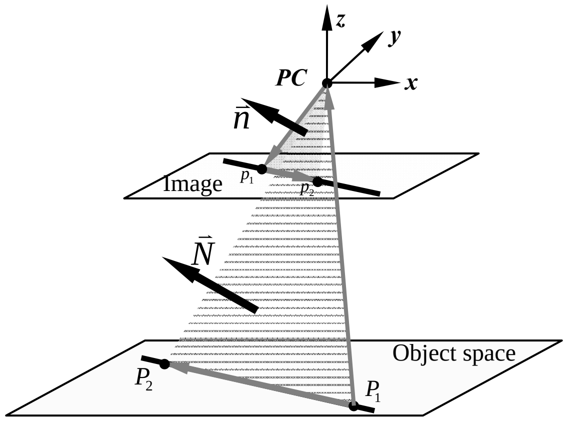

Figure 1. Interpretation plane and normal vectors.

The interpretation plane contains the straight line in the object space, the projected straight line in the image space, and the perspective centre of the camera (**PC**) (Fig. 1).

In the original equivalent plane model the vector ***n***, normal to the interpretation plane in the image space, was expressed was a function of the focal length and the ***θ-ρ*** parameters of the straight line in the photogrammetric reference system. In order to include the lens distortion parameters and the coordinates of the principal point in the model, the straight line in the image space have to be expressed by the coordinates of its endpoints. Considering the Conrady-Brown lens distortion model the coordinates of an image point in the photogrammetric reference system can be given by eq. 1.

$$\begin{aligned} x &= x' - x_0 + \bar{x}(K_1 r^2 + K_2 r^4 + K_3 r^6) + P_1(r^2 + 2\bar{x}^2) + 2P_2\overline{xy} \\ y &= y' - y_0 + \bar{y}(K_1 r^2 + K_2 r^4 + K_3 r^6) + P_2(r^2 + 2\bar{y}^2) + 2P_1\overline{xy} \end{aligned} \quad (1)$$

where:

x, y are the photogrametric coordinates;
x', y' are the observed point coordinates related to a centered image system;
K_1, K_2, K_3 are the radial lens distortion coefficients;
P_1, P_2 are the decentering distortion coefficients;
x_0, y_0 are the coordinates of the principal point;
$\bar{x} = x' - x_0$;
$\bar{y} = y' - y_0$;
$r = \sqrt{\bar{x} + \bar{y}}$.

The vector ***n***, normal to the interpretation plane in the image space, can then be written as the following vector product:

$$\vec{n} = \begin{bmatrix} \Delta x_{12} \\ \Delta y_{12} \\ 0 \end{bmatrix} \wedge \begin{bmatrix} x_1 \\ y_1 \\ -c \end{bmatrix} = \begin{bmatrix} -c\Delta y_{12} \\ c\Delta x_{12} \\ x_2 y_1 - x_1 y_2 \end{bmatrix} \quad (2)$$

where:
$\Delta x_{12} = x_2 - x_1$;
$\Delta y_{12} = y_2 - y_1$
c is the camera focal length.

The vector ***N***, normal to the interpretation plane in the object space, is defined by the vector product of the direction vector (***P_2 - P_1***) of the straight line and the vector difference (***PC - P_1***) (See Fig. 1).

$$\vec{N} = \begin{bmatrix} \Delta X_{12} \\ \Delta Y_{12} \\ \Delta Z_{12} \end{bmatrix} \wedge \begin{bmatrix} \Delta X_{01} \\ \Delta Y_{01} \\ \Delta Z_{01} \end{bmatrix} = \begin{bmatrix} \Delta Y_{12}\Delta Z_{01} - \Delta Y_{01}\Delta Z_{12} \\ \Delta X_{01}\Delta Z_{12} - \Delta X_{12}\Delta Z_{01} \\ \Delta X_{12}\Delta Y_{01} - \Delta X_{01}\Delta Y_{12} \end{bmatrix} \quad (3)$$

where:
$\Delta X_{12} = X_2 - X_1$;
$\Delta Y_{12} = Y_2 - Y_1$;
$\Delta Z_{12} = Z_2 - Z_1$;
$\Delta X_{01} = X_1 - X_0$;
$\Delta Y_{01} = Y_1 - Y_0$;
$\Delta Z_{01} = Z_1 - Z_0$.

X_0*, *Y_0*, *Z_0 are the coordinates of the Perspective Centre (***PC***) of the camera related to the object reference system; ***X_1*, *Y_1*, *Z_1*** and ***X_2*, *Y_2*, *Z_2*** are the 3D coordinates of two endpoints in the object straight line.

Multiplying vector ***N*** by the rotation matrix ***R*** eliminates angular differences between the object and the image reference systems and results in a vector normal to the interpretation plane in object space that has the same orientation as vector ***n***, normal to the interpretation plane in the image space, but different in magnitude.

$$\mathrm{R}.\vec{\mathrm{N}} = \lambda.\vec{\mathrm{n}} \quad (4)$$

Where λ is a scale factor and ***R*** is the rotation matrix defined by the sequence *$R_z(\kappa).R_y(\varphi).R_z(\omega)$* of rotations.

$$R_{\omega,\phi,\kappa} \begin{bmatrix} \Delta Y_{12}\Delta Z_{01} - \Delta Y_{01}\Delta Z_{12} \\ \Delta X_{01}\Delta Z_{12} - \Delta X_{12}\Delta Z_{01} \\ \Delta X_{12}\Delta Y_{01} - \Delta X_{01}\Delta Y_{12} \end{bmatrix} = \lambda \begin{bmatrix} -c\Delta y_{12} \\ c\Delta x_{12} \\ x_2 y_1 - x_1 y_2 \end{bmatrix} \quad (5)$$

Expanding (5) gives:

$$\begin{aligned} r_{11}N_1 + r_{12}N_2 + r_{13}N_3 &= -\lambda c\Delta y_{12} \\ r_{21}N_1 + r_{22}N_2 + r_{23}N_3 &= \lambda c\Delta x_{12} \\ r_{31}N_1 + r_{32}N_2 + r_{33}N_3 &= \lambda(x_2 y_1 - x_1 y_2) \end{aligned} \quad (6)$$

In order to eliminate the element λ the first and the second equations of (6) are divided by the third one, giving:

$$\begin{aligned} &(x_2y_1 - x_1y_2)(r_{11}N_1 + r_{12}N_2 + r_{13}N_3) + c\Delta y_{12}(r_{31}N_1 + r_{32}N_2 + r_{33}N_3) = 0 \\ &(x_2y_1 - x_1y_2)(r_{21}N_1 + r_{22}N_2 + r_{23}N_3) - c\Delta x_{12}(r_{31}N_1 + r_{32}N_2 + r_{33}N_3) = 0 \end{aligned} \quad (7)$$

By substituting the distortion equations (1) in (7) gives the final model.

In the original equivalent planes model (Tommaselli and Tozzi, 1996) there was a need to explicitly isolate the observations, in order to use the Iterated Extended Kalman Filtering, in a sequential approach. Due to this need and to avoid divisions by zero, two groups of equations were derived, depending on the line orientation in the image. In the current case, a different strategy for deriving the model was used, because a more general estimation model was adopted (Mikhail and Ackerman, 1976), avoiding divisions and, therefore, generating just two equations suitable for lines of any orientation.

Although the model has two endpoints coordinates as observations, it is not required that these image endpoints should be correspondent to endpoints in the object space, thus preserving one the main advantages of the line based models. A unified method of adjustment with weighted constraints to the parameters was used for parameter estimation giving a high degree of flexibility (Mikhail and Ackerman, 1976). The estimation method was implemented in C++ language. A conventional bundle adjustment based on collinearity equations was also implemented in order to be used as a reference method. A linear feature extraction method aiming at sub-pixel precision was implemented based on the analysis of a profile perpendicular to the edge orientation (Jain et al, 1995)

3. EXPERIMENTS AND RESULTS

The proposed method was implemented in C++ language and tested with simulated and real data. In this section some experiments and results are presented and discussed.

3.1 Experiments with simulated data

In order to perform the experiments with simulated data a set of 43 straight lines and 43 points was defined in the object space (Fig. 2).

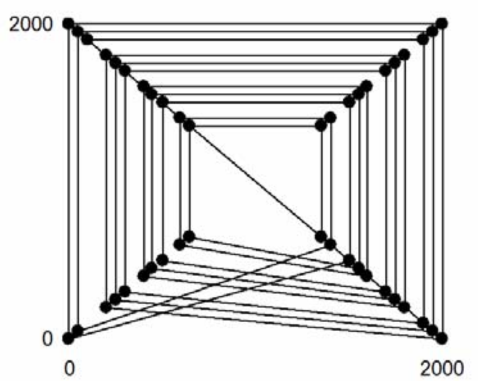

Figure 2. Simulated straight lines in the object space (units in mm).

The interior orientation parameters and the effects of systematic errors in one of the image corners (*x=17mm, y=17mm*) are presented in Table 1, supposing a digital camera with a 35x35mm frame.

Figure 3 depicts the six generated images with different exterior orientation in order to guarantee a suitable level of convergence.

IO parameters	Simulated Value	Effect of systematic errors in the image coordinates of point in the image corner	
		x	*y*
c (mm)	35.0	-	-
x_0 *(mm)*	0.2	0.2	-
x_0 *(mm)*	0.3	-	0.3
K_1 (mm^{-2})	1.0E-05	0.098	0.098
K_2 (mm^{-4})	2.0E-09	0.011	0.011
K_3 (mm^{-6})	5.0E-12	6.83E-04	6.83E-04
P_1 (mm^{-1})	2.0E-05	0.023	0.035
P_2 (mm^{-1})	3.0E-05	0.012	0.017

Table 1. Simulated Interior Orientation Parameter (IOP) and the effects of systematic errors in an image point on the image corner

Approximated values for the unknowns used in the estimation process were given with errors of 0.3 rad for rotations and 150 to 250 mm for the PC coordinates. Initial values for the interior orientation parameters were also considered with errors: 45 mm for the focal length, zero for the coordinates of the principal point and for the lens distortion parameters. The coordinates of the endpoints of the straight lines in the object space were supposed to have an error of 0.25 mm in its components.

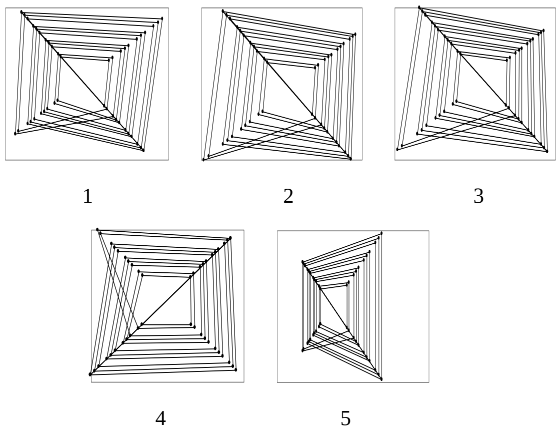

Figure 3. Simulated images.

Using this data set, several experiments were performed varying the following data:

- the magnitude of the random errors introduced in the image coordinates;
- Camera position and orientation;
- Configuration of the straight lines in the object space.

The same data set was used in experiments using conventional calibration with bundle adjustment in order to compare the results. Due to the lack of space just one group of experiments will be presented in this paper.

Experiments with different random errors

Experiments with random errors with standard deviations of ***1 μm***, ***5 μm*** and ***10 μm*** were accomplished. In all cases both IOP and EOP were correctly estimated, as can be seen comparing table 2 and table 1. The results were tested with a χ^2 test with a significance level of 0.95 and were accepted. Table 2 presents the IOP computed using simulated data with a standard deviation of ***5 μm*** in the random errors.

IOP	Method using straight lines		Bundle Method with points	
	Estimated Value	**Standard Deviation**	**Estimated Value**	**Standard Deviation**
c (mm)	34.96	2.4E-02	34.97	2.3E-02
x_0 *(mm)*	0.19	1.6E-02	0.19	1.7E-02
x_0 *(mm)*	0.31	1.5E-02	0.31	1.6E-02
K_1 (mm^{-2})	8.1E-06	3.3E-06	9.7E-06	3.2E-06
K_2 (mm^{-4})	7.6E-09	1.0E-08	2.7E-09	9.9E-09
K_3 (mm^{-6})	1.3E-12	1.0E-11	5.5E-12	9.7E-12
P_1 (mm^{-1})	2.5E-05	3.8E-06	2.6E-05	4.0E-06
P_2 (mm^{-1})	2.6E-05	3.4E-06	2.5E-05	4.0E-06

Table 2 – Estimated IOP and their standard deviations using both straight lines and points with a standard deviation of *5 µm* in the random errors.

In order to assess the performance of the proposed method the IOP were also estimated by conventional self calibrating bundle adjustment. Similar redundancies were achieved in both procedures (225 image straight lines with 412 degrees of freedom, and 215 image points with 392 degrees of freedom).

Analyzing the results presented in table 2 it can be verified that the values estimated from both methods are similar and are within the estimated standard deviations. Similar results were also achieved with other levels of random errors (***1 µm, 10 µm***) It is also clear that with that level of noise (***5 µm***) the coefficients K_2 e K_3 can be neglected.

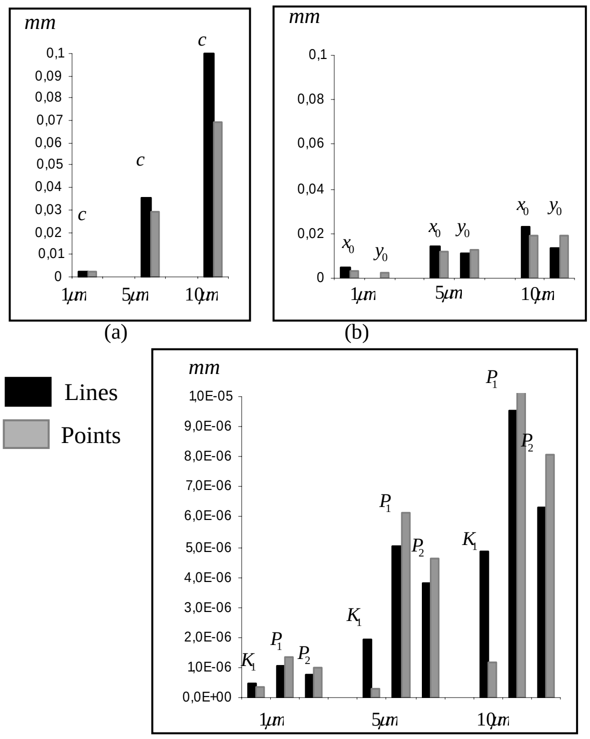

Figure 4. True errors in the IOP considering different levels of random errors in the observations: (a) errors in the focal length; (b) errors in the coordinates of the principal point; (c) errors in the lens distortion coefficients.

A ***t*** *Student* statistical test was then applied and it was verified that the estimated standard deviations are compatible with the true errors at a level of significance of 0.05. If the magnitude of the random error is increased the quality of the estimated standard deviation decreases, but this also happened with the conventional self calibrating bundle adjustment with points. This is probable caused error propagation an over-parametrization, because some high order of lens distortion can only be recovered with high quality image observations, but the verification of this hypothesis is left for future work.

Figure 4 depicts true errors (discrepancies between the estimated and the true values) in the IOP considering the experiments in which several magnitudes of random errors in the observations were tested. As expected, true errors increases proportionally to the standard deviation of the random errors. Some parameters were better estimated with the straight line method (y_0, P_1, P_2) whilst others presented better results with the conventional bundle method, but it cannot be stated with confidence which method is better with the performed experiments.

3.2 Experiments with real data

A close range test field, 2mx2m, with 16 black tiles (32 edges) over a white wall was built. The coordinates of two endpoints in each edge were measured with a metallic scale, with an uncertainty around 1mm.

Five images (two of them convergent and one with 90^o rotation around z axis) were taken with a Sony F828 digital camera (8 megapixel) with a 50 mm focal length focused to the infinite. Two end points for each straight line were measured in these experiments using a subpixel extraction technique (Jain et al, 1995).

In order to assess the accuracy of the proposed method with real data the bundle method was also applied to enable the comparison of the estimated parameters for both methods. Control points in the object space were computed as the intersection of two edges and the measurement of its homologous in the image was performed by visual screen pointing.

It is important to note that when using the straight line method there no need for point to point correspondence.

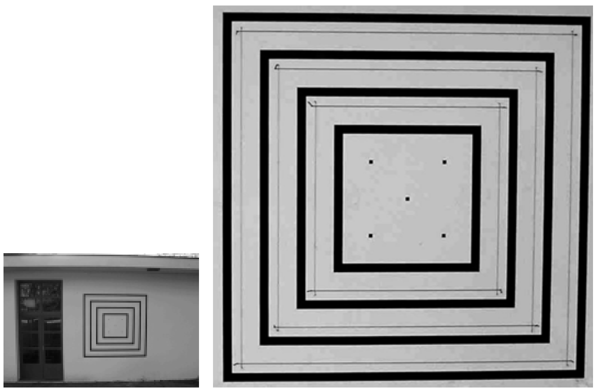

Figure 5. Test field with straight lines.

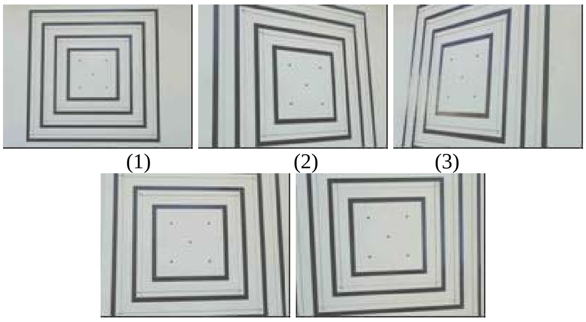

Figure 6. Images used in the experiments with real data.

In both group of experiments (with straight lines and with points) the following IOP were considered: focal length, principal point coordinates and first order radial lens distortion coefficient (K_1). The decentering distortion coefficients and the higher order radial lens distortion coefficients were neglected after some preliminary experiments showing that their magnitudes were small and incompatible with the quality of the observed image coordinates.

Table 3 presents IOP and the EOP of the first image, which were estimated using straight lines; in the last column the discrepancies related to the same parameters computed using self calibrating bundle method are presented. There were 200 degrees of freedom for the experiment using straight lines and 184 for the experiments using bundle adjustment with points.

In general, the results have shown that the developed model is comparable to the conventional bundle method using control points. However, the EOP and the coordinates of the principal points presented higher discrepancies whilst the other parameters had similar results. These discrepancies can be explained by the correlations between parameters and by narrow coverage angle of the camera. Other problem is the correlation between the observations when using straight lines. A detailed analysis of the correlations between parameters and observations is recommended as part of a future work.

IOP and EOP	Calibration using straight lines		Discrepancies with reference to the bundle method
	Estimated Value	Estimated Standard Deviation	
c (mm)	51.991	0.121	-0.325
x_0 (mm)	-0.125	0.010	-0.482
y_0 (mm)	0.124	0.058	0.018
K_1 (mm^{-2})	-5.94E-06	1.56E-06	5.11E-07
κ (rad)	-2.95E-03	2.39E-04	-6.14E-03
φ (rad)	3.17E-02	1.22E-03	1.90E-03
ω (rad)	8.61E-02	1.32E-03	-4.80E-04
X_0 (mm)	1196.519	5.244	-30.755
Y_0 (mm)	626.915	5.157	5.769
Z_0 (mm)	4130.852	9.397	-28.408

Table 3 – IOP and EOP estimated using straight lines and its discrepancies with those computed using bundle method.

In order to assess the effect of the estimated IOP in the photogrammetric intersection when using different methods (straight line based method or conventional bundle method) a two photo resection was computed using 6 control points (fig. 7). Then, coordinates of 15 check points were estimated using photogrammetric intersection.

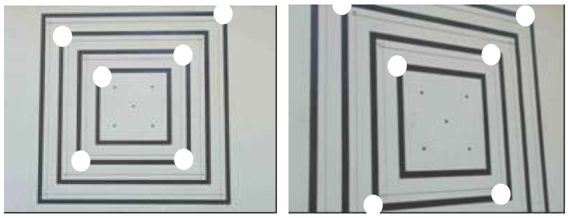

Figure 7. Images used for checking the computed IOP; points highlighted were used as control points.

The average of the discrepancies between the photogrammetric derived coordinates and the field true was **0.597±1.591 mm** when using **IOP** computed with the **straight line** method and **0.530±1.512** mm when using the IOP generated by bundle adjustment with points. The RMS of the discrepancies in XYZ coordinates were **1.702mm** for the **straight line** based experiment and **1.604mm** for the **point based** one.

4. CONCLUSIONS

The results obtained with the proposed method are statistically comparable with the conventional self calibrating bundle method with points, although the bundle method provided in the experiment with real data slightly better results. It is important to remember, however, that in this experiment the authors still did not take advantage of the potential redundancy provided by the straight lines. For each straight line only two equations were generated although there is a potential for using several pairs of equations. The potential for multiple line measurements and its impact in the final results is still a topic to be studied in future work. Another difference in comparison with the simulated experiments is that no oblique lines were used in the real test field.

It still worth to note the advantages of line based calibration: flexibility; no need for point to point correspondence; lines can be extracted with subpixel accuracy. A topic of interest is the combination of points and lines, in order to get together the rigidity of points and the redundancy provided by lines.

References

Brown, D., 1971. Close-Range Camera Calibration, *Photogrammetric Engineering and Remote Sensing*, 37(8): 855-866.

Ethrog, U., 1984. Non-metric camera calibration and photo orientation using parallel and perpendicular lines of the photographed objects, *Photogrammetria*, 39: 13-22.

Habib, A.F.; Morgan, M.; Lee, Y.R., 2002. Bundle Adjustment with self-calibration using straight lines. *Photogrammetric Record*, Vol. 17, pp. 635-650.

Habib, A.F.; Lin, H.T.; Morgan, M., 2003. Autonomous space resection using point- and line-based representation of free-form control linear features. *Photogrammetric Record*, Vol. 18, pp. 244-258.

Haralick, Robert M., 1989. Determining camera parameters from the perspective projection of a rectangle, *Pattern Recognition,* 22(3): 225-230.

Jain, R.; Kasturi, R.; Schunck, B. G. 1995**.** *Machine Vision***.** New York: McGraw-Hill International Editions, Computer Science Series.

Lee, R.; Lu, P.C and Tsai, W.H., 1990. Robot location using single views of retangular shapes, *Photogrammetric Engineering and Remote Sensing*, Vol. 56, No. 2, pp. 231-238.

Lenz, R. K. and Tsai, R. Y., 1988. Techniques for Calibration of the Scale Factor and Image Center for High Accuracy 3-D Machine Vision Metrology, *IEEE Transactions on PAMI.* Vol. 10, No. 5.

Liu, Y.; Huang, T. S. And Faugeras, O. D., 1990. Determination of camera location from 2-D to 3-D line and point correspondences, *IEEE Transactions on PAMI*, Vol. 12, No. 1, pp. 28-37.

Lugnani, J.B., 1980. *Using digital entities as control*, PhD Dissertation, Department of Surveying Engineering, The University of New Brunswick, Fredericton, 159 p.

Masry, S.E., 1981. Digital Mapping Using Entities: A new concept, *Photogrammetric Engineering and Remote Sensing*, 48(11).

Mikhail, E. M., Ackerman, F. 1976. *Observations and Least Squares*. IEP Series in Civil Engineering, New York.

Mulawa, D. C. and Mikhail, E. M., 1988. Photogrammetric treatment of linear features, In: *Proceedings of the International Society for Photogrammetry and Remote Sensing*, 1988, Kyoto, Commission III, pp. 383-393.

Prescott, B. and Mclean G.I., 1997. Line Base Correction of Radial Lens Distortion. *Graphical Models and Image Processing.* Vol. 59, No 1, pp. 39-47.

Quan, L. and Kanade, T., 1997. Affine Structure from Line Correspondences With Uncalibrated Affine Cameras. *IEEE Transactions on PAMI.* Vol. 19, No. 8, pp. 834-845.

Schenk, T., 2004. From point-based to feature-based aerial triangulation. ISPRS Journal of Photogrammetry & Remote Sensing, Vol. 58, pp. 315– 329.

Telles, S. S.; Tommaselli, A. M. G., 2002. Calibração de câmaras digitais usando linhas retas*. Série em Ciências Geodésicas.* Curitiba, Vol. 2, pp. 289-314.

Tommaselli, A.M.G. and Lugnani, J. B., 1988. An alternative mathematical model to the collinearity equation using straight features, In: *Proceedings of the International Society for Photogrammetry and Remote Sensing*, Kyoto, Commission III, pp. 765-774.

Tommaselli, A.M.G. and Tozzi, C. L., 1996. A recursive approach to Space Resection using straight lines, *Photogrammetric Engineering and Remote Sensing,* Vol. 62, No 1, pp. 57-66.

Tommaselli, A.M.G., 2000. Orientation and Reconstruction of Close-Range Images using Lines. In: *International Archives of Photogrammetry and Remote Sensing*. Amsterdam. V. XXXIII, pp. 838-845.

Wang, L.L. and Tsai, W.H., 1990. Computing camera parameters using vanishing-line information from a rectangular parallelepiped, *Machine Vision and Applications*, Vol.3, pp.12.

Acknowledgements

The authors would like to acknowledge the support of CNPQ (Conselho Nacional de Desenvolvimento Cientifico e Tecnológico), with a Master Scholarship. The authors are also thankfull to Dr. Maurício Galo whose program CC (Camera Calibration) was used to check the preliminary results and in the two photo resection.