

# On the Use of Inertial/GPS Velocity Control in Sensor Calibration and Orientation

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## Abstract

Since the 1980's and 1990's, GPS and INS/GPS data, respectively, have been used to support sensor calibration and orientation of kinematic primary data acquisition systems. Aerial control position and attitude observations, well-known to the photogrammetric community, are used for Direct Sensor Orientation (DSO) and combined with the traditional aerial triangulation image observations to reduce ground control and mitigate the old constraints on the shape of blocks in Integrated Sensor Orientation (ISO).

In the past two years, at the Institute of Geomatics (IG), research has been conducted to improve the reliability and robustness of the mathematical models for aerial position and attitude control. This effort has already led to new models for relative position and attitude control. In this paper, the authors introduce models for the temporal calibration of multi-sensor systems. For this purpose, the INS/GPS-derived velocities are used as velocity control information. To the best knowledge of the authors, this information has not yet been used for ISO.

The paper is based on two main ideas. First, that INS/GPS data include linear and angular velocities, not only position and attitude. Second, that the sensor calibration and orientation problem is not just a 3D spatial problem, but a 4D spatio-temporal problem and that, as experience has shown, the temporal dimension must be taken into account. This paper describes the proposed models in which the velocities are included to calibrate the time error of the various sensors of a multi-sensor system with respect to the accurate and precise GPS time reference frame.

The authors believe that the addition of the velocities to calibrate time could make the difference between “good” and “excellent” results at no additional cost as the newly used data –the velocities– are already available from the INS/GPS processing.

## 1 Introduction

The use of Global Positioning System (GPS) and Inertial Navigation System (INS) technologies to support remote sensing acquisition missions is now widespread. These technologies have been winning followers ([9] and [11]) since the 1980's and 1990's and, for various reasons, the majority of mobile mapping data acquisition systems are equipped with an Inertial Measurement Unit (IMU) and at least one GPS receiver antenna.

Two main orientation and calibration procedures have been established over the past twenty years: Direct Sensor Orientation (DSO) and Integrated Sensor Orientation (ISO). Both are based on the transfer of INS/GPS-derived position and attitude to the sensor position and attitude. ISO is an orientation and calibration procedure that optimally estimates the multi-sensor system parameters in the sense of least-squares (see [1], [2], [4]) while DSO is an orientation procedure that directly transfers the INS/GPS position and attitude to the sensor position and attitude (see [11]). In the case of DSO, depending on the required precision, a prior ISO calibration step is usually unavoidable (see [3], [5] and [7]).

Generally speaking, geomatic system development is taking place in two different directions: better performance and lower cost. In principle, without the introduction of new technology, the two directions diverge. Users would like to have it both ways (who wouldn't?): best performance at the lowest cost, the ultimate “consumer dream.” In parallel, geoinformation has become a fundamental infrastructure in a society that needs to keep it up-to-date at an affordable cost. The net result is that the “consumer dream” is becoming a “citizen demand,” and the “citizen demand” a compelling need for faster and cheaper production. We might wonder if traditional orientation and calibration procedures are simple, reliable and robust enough to tackle this contradictory situation. Our contribution to the above situation and discussion is the proposal of new sensor models, which are useful for the best and worst cases, without any extra cost, because all the required data are already available.

In this context, the paper presents the improvement of the position and attitude aerial control mathematical functional models based on two key points:

- Since the IMU and the sensor are usually rigidly assembled, the relative orientation of the IMU between two epochs is the same as the relative orientation of the sensor between the same two epochs.
- The orientation and calibration problem is not a 3D  $(x, y, z)$ -problem but a 4D  $(t, x, y, z)$ -problem. The temporal calibration parameters can be estimated using velocities from INS/GPS data.

This paper is organized in four main sections. The reliability and robustness concepts are reviewed in the next section. The motivations and principles behind the improvement of the multi-sensor system modelling are presented thereafter. The last two sections develop the proposed mathematical functional models for one or two epoch-data and for temporal calibration with INS/GPS-derived velocities.

## 2 Reliability and Robustness

Tremendous technological progress has taken place in the geomatic world over the last two decades. The geomatic community has incorporated new technology (GPS and INS), developed new sensors (Airborne Laser Scanner and Airborne Radar) and improved the design of existing ones (digital cameras in terrestrial, aerial and satellite platforms). Further, particularly on the hardware level, system complexity and cost have broadened. Primary data acquisition can now be “better” performed with large format digital cameras equipped with geodetic grade GPS receivers and navigation grade IMUs, or with lightweight medium format cameras equipped with low-cost GPS receivers and Micro-Electro-Mechanical Systems (MEMS) based IMUs. However, even off-the-shelf mass-market hardware devices with their high price-performance have limitations in terms of precision, accuracy and reliability. Without appropriate modelling and its results, better algorithms and data processing software, the “consumer dream” is unlikely to come true.

As mentioned earlier, mapping companies face the challenge of processing large amounts of data under higher time pressure and with tighter budgets than in the past. Moreover, and for reasons beyond the scope of a technical paper, they often have no choice than to do so with less trained staff. In such circumstances, automation and robustness are becoming more relevant beyond meeting technical specifications. Research teams can meet the challenge by providing simpler, more reliable and more robust strategies.

In our opinion, simplicity will be the result of maturity; i.e., of correct abstractions and models. Above all, it is simplicity at the user –not the developer– end that matters: correct modelling and concepts make users’ lives easier; simple modelling and concepts just make developers’ lives easier.

Reliability and robustness have a manifold of domain dependent interpretations.

According to the Institute of Electrical and Electronics Engineers (IEEE), reliability is the ability of a system or component to perform its required functions under stated conditions for a specified period of time (see [6]). In the context of the geodetic and photogrammetric communities, reliability is the ability to detect gross errors and not be too sensitive to undetected gross errors.

According to the IEEE, robustness is the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environment conditions (see [6]). In the context of the geodetic and photogrammetric communities, robustness is the ability to provide correct results even in the presence of gross errors. (In the context of the statistics community, a robust strategy can be defined as a strategy with the following behaviour: if the data are not flawed, the robust strategy will deliver correct results —within specifications, although under these ideal circumstances, a classical strategy will deliver better results. But if some data are wrong, the robust strategy will still deliver correct results —worse than the results obtained under ideal circumstances, but within the specifications— whereas a classical strategy will fail.)

In the authors’ opinion, there is room for improvement in sensor and related modelling to make our strategies simpler, more reliable and more robust. This paper presents a contribution in this direction that is based on the use of available INS/GPS data.

## 3 On the Use of Available INS/GPS Data

In the traditional ISO procedure, the orientation and calibration models relate the camera<sup>1</sup> measurements (image coordinates), ground control points, and INS/GPS-derived position and attitude data with the unknown system parameters (ground points, exterior orientation parameters, lab-calibration and self-calibration parameters, eccentricity vectors, IMU-to-sensor relative orientation, etc.) for each epoch. We will refer to these models as absolute orientation models.

The models described in this paper can be regarded as extensions of the classical absolute orientation models.

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<sup>1</sup>Until now, we have been using the generic word “sensor.” In the next sections, we will focus on photogrammetric cameras. However, the proposed models can either be applied or easily adapted to any other remote sensing sensor.

### 3.1 From INS/GPS Data to Camera Orientation

Based on the fact that the relative orientation of the INS/GPS system between two epochs is the same as the sensor relative orientation between the same two epochs (assuming that the sensor and IMU are rigidly assembled), the traditional ISO procedure can be extended with orientation and calibration models that relate camera measurements (image coordinates), ground control points, and INS/GPS-derived position and attitude measurements with unknown system parameters (ground points, exterior orientation parameters, lab-calibration and self-calibration parameters, eccentricity vector, etc.) every two consecutive epochs. These mathematical models are called relative orientation models in this paper.

### 3.2 From Position and Attitude to Position, Attitude and Velocity

Based on the fact that an INS/GPS system does not only provide position and attitude, but also provides velocity, the traditional ISO procedure can be extended with temporal calibration models that relate the camera measurements (image coordinates), ground control points, and INS/GPS-derived position, velocity and attitude measurements with the unknown system parameters (ground points, exterior orientation parameters, lab-calibration and self-calibration parameters, eccentricity vector, etc.) and multi-sensor time synchronization parameters. In the paper these mathematical models are called temporal calibration models.

## 4 Traditional 3D Problem: Absolute and Relative Orientation

The classical procedure is to transfer INS/GPS-derived orientation to image orientation, direct or indirect, with absolute orientation models and, in the case of attitude control, through rotation matrix reparameterization operations. This procedure can be improved with the proposed relative position and aerial control models (see [10]) because:

- Steps to transform the INS/GPS traditional attitude parameterization  $(\psi, \vartheta, \gamma)$  [heading, pitch, roll] into the photogrammetric traditional one  $(\omega, \phi, \kappa)$  are unnecessary.
- The IMU-to-sensor relative orientation [boresight] calibration parameter is eliminated.
- GPS trajectory errors due to undetected cycle slips easily can be better detected and removed.

Just for the sake of completeness, the absolute position and attitude aerial control mathematical functional models are presented in the next section. After this, the relative position and attitude aerial control mathematical functional models are developed.

### 4.1 Absolute Position and Attitude Aerial Control Mathematical Functional Models

$$\begin{pmatrix} x + r_x \\ y + r_y \\ z + r_z \end{pmatrix}^l = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^l + R_c^l(\omega, \varphi, \kappa) \cdot \begin{pmatrix} a_x \\ a_y \\ a_z + n \end{pmatrix}^c + \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}^l, \quad (1)$$

$$R_c^l(\omega, \varphi, \kappa) = R_l^l \cdot R_b^{\bar{l}}(\psi + r_\psi, \vartheta + r_\vartheta, \gamma + r_\gamma) \cdot R_b^{\bar{b}} \cdot R_c^b(\epsilon_x, \epsilon_y, \epsilon_z) \quad (2)$$

are the mathematical functional models for the absolute position (equation 1) and attitude (equation 2) aerial control where the involved reference frames are:

- $l$  Cartesian local terrestrial frame (east-north-up),
- $\bar{l}$  Cartesian local terrestrial frame (north-east-down),
- $b$  IMU instrumental frame (forward-left-up),
- $\bar{b}$  IMU instrumental frame (forward-right-down), and
- $c$  camera instrumental frame.

For the sake of simplicity, we will write  $(a, b, c)^f$  instead of the rigorous mathematical formulation that is  $\left[ (a, b, c)^f \right]^T$  to represent the vector  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}^f$ .

The observables (measurements) are represented by:

- $(x, y, z)^l$  GPS- or INS/GPS-derived position, and

- $(\psi, \vartheta, \gamma)$  traditional [heading, pitch, roll] Euler angles which parameterize the  $R_{\bar{b}}^{\bar{l}}$  rotation matrix from the instrumental  $\bar{b}$  reference frame to the  $\bar{l}$  reference frame.

Here, and for the rest of residuals, we will write  $r_\alpha$  to indicate the residual of the observable  $\alpha$ . Therefore,  $(r_x, r_y, r_z)^l$  are the GPS- or INS/GPS-derived position residuals and  $(r_\psi, r_\vartheta, r_\gamma)$  are the [heading, pitch, roll] Euler angles residuals.

The parameters (unknowns) involved in the models are:

- $(X, Y, Z)^l$  camera projection centre,
- $(\omega, \varphi, \kappa)$  traditional Euler angles which parameterize the  $R_c^l$  rotation matrix from the instrumental  $c$  reference frame to the  $l$  reference frame,
- $(a_x, a_y, a_z)^c$  eccentricity vector from the camera projection centre to the GPS receiver antenna,
- $(s_x, s_y, s_z)^l$  GPS positioning errors, and
- $R_c^b(\epsilon_x, \epsilon_y, \epsilon_z)$  IMU-to-camera relative orientation [boresight] calibration parameter.

And the known constants (instrumental and rotation matrices) are:

- $n$  camera nodal distance,
- $R_{\bar{l}}^l = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ , and
- $R_{\bar{b}}^{\bar{l}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ .

#### 4.2 Relative Position and Attitude Aerial Control Mathematical Functional Models

The mathematical functional models for the relative position and attitude aerial control are

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}^l - \begin{pmatrix} x_2 + r_{x_2} \\ y_2 + r_{y_2} \\ z_2 + r_{z_2} \end{pmatrix}^l = \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}^l - \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}^l + (R_c^l(\omega_1, \varphi_1, \kappa_1) - R_c^l(\omega_2, \varphi_2, \kappa_2)) \cdot \begin{pmatrix} a_x \\ a_y \\ a_z + n \end{pmatrix}^c, \quad (3)$$

$$R_c^l(\omega_1, \varphi_1, \kappa_1) \cdot R_{\bar{l}}^c(\omega_2, \varphi_2, \kappa_2) = R_{\bar{l}}^l \cdot R_{\bar{b}}^{\bar{l}}(\psi_1, \vartheta_1, \gamma_1) \cdot R_{\bar{l}}^{\bar{b}}(\psi_2 + r_{\psi_2}, \vartheta_2 + r_{\vartheta_2}, \gamma_2 + r_{\gamma_2}) \cdot R_{\bar{l}}^{\bar{b}} \quad (4)$$

respectively, where the involved reference frames are the same as in the previous section.

The observables (measurements) are represented by:

- $(x_2, y_2, z_2)^l$  GPS- or INS/GPS-derived position at epoch  $t_2$ , and
- $(\psi_2, \vartheta_2, \gamma_2)$  Euler angles which parameterize the  $R_{\bar{b}}^{\bar{l}}$  rotation matrix from the instrumental  $\bar{b}$  reference frame to the  $\bar{l}$  reference frame at epoch  $t_2$ .

The parameters (unknowns) involved in the models are:

- $(X_1, Y_1, Z_1)^l$  camera projection centre at epoch  $t_1$ ,
- $(\omega_1, \varphi_1, \kappa_1)$  Euler angles which parameterize the  $R_c^l$  rotation matrix from the instrumental  $c$  reference frame to the  $l$  reference frame at epoch  $t_1$ ,
- $(X_2, Y_2, Z_2)^l$  camera projection centre at epoch  $t_2$ ,
- $(\omega_2, \varphi_2, \kappa_2)$  Euler angles which parameterize the  $R_c^l$  rotation matrix from the instrumental  $c$  reference frame to the  $l$  reference frame at epoch  $t_2$ , and
- $(a_x, a_y, a_z)^c$  (see 4.1).

And the known observational auxiliary values and constants (instrumental and rotation matrices) are:

- $n$  and  $R_{\bar{l}}^l$  (see 4.1),
- $(x_1, y_1, z_1)^l$  GPS- or INS/GPS-derived position at epoch  $t_1$ , and
- $(\psi_1, \vartheta_1, \gamma_1)$  Euler angles which parameterize the  $R_{\bar{b}}^{\bar{l}}$  rotation matrix from the instrumental  $\bar{b}$  reference frame to the  $\bar{l}$  reference frame at epoch  $t_1$ .

In the relative control formulation, in this section as well as in section 5.2, the pairs of observational data sets (for the left and right images) have been “transformed” into a constant auxiliary data set (epoch named 1) and an observational data set (epoch named 2) for numerical related purposes. This transformation has no impact, neither on the functional nor the stochastic model properties.

## 5 Actual 4D Problem: Spatio-temporal Calibration

The traditional formulation of the sensor orientation and calibration problem does not take the temporal dimension into account. However, spatio-temporal calibration is nothing particularly new in geomatics. For example, the GPS range equations model the time synchronization error between the GPS receiver clock and the GPS satellite clocks. Therefore, it is quite natural that spatio-temporal functional models be developed for sensor orientation and calibration. The following formulas are realizations of this idea; more specifically they are functional models, with absolute and relative control data, for sensor orientation with temporal calibration parameters. The proposed models have, among others, two advantages:

- The time inconsistencies between the different instrumental time reference frames of the multi-sensor system can be de-correlated from spatial errors.
- GPS or INS/GPS velocity control can be used to perform the temporal calibration in low cost hardware systems (where the development of tailored precise time synchronization electronics may be too expensive).

### 5.1 Absolute Position, Velocity, and Attitude Aerial Control Mathematical Functional Models

The mathematical functional models for the absolute position, velocity and attitude aerial control are:

$$\begin{pmatrix} x + r_x \\ y + r_y \\ z + r_z \end{pmatrix}^l = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^l + R_c^l(\omega, \varphi, \kappa) \cdot \begin{pmatrix} a_x \\ a_y \\ a_z + n \end{pmatrix}^c + \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}^l - \begin{pmatrix} v_x + r_{v_x} \\ v_y + r_{v_y} \\ v_z + r_{v_z} \end{pmatrix}^l \cdot \delta t \quad (5)$$

$$R_c^l(\omega, \varphi, \kappa) = R_{\bar{l}}^l \cdot \left[ R_{\bar{b}}^{\bar{l}}(\psi + r_\psi, \vartheta + r_\vartheta, \gamma + r_\gamma) + \dot{R}_{\bar{b}}^{\bar{l}}(\psi + r_\psi, \vartheta + r_\vartheta, \gamma + r_\gamma) \cdot \delta t \right] \cdot R_{\bar{b}}^{\bar{b}} \cdot R_c^b(\epsilon_x, \epsilon_y, \epsilon_z) \quad (6)$$

where

$$\dot{R}_{\bar{b}}^{\bar{l}}(\psi + r_\psi, \vartheta + r_\vartheta, \gamma + r_\gamma) = R_{\bar{b}}^{\bar{l}}(\psi + r_\psi, \vartheta + r_\vartheta, \gamma + r_\gamma) \cdot \left[ \Omega_{i\bar{b}}^{\bar{b}}(\omega_x, \omega_y, \omega_z) - \Omega_{i\bar{l}}^{\bar{b}}(\lambda, \phi, \dot{\lambda}, \dot{\phi}, \omega_e) \right]$$

The involved reference frames are the same as in the previous sections (see 4.1) and the inertial  $i$  reference frame.

The observables (measurements) are represented by:

- $(x, y, z)^l$  and  $(\psi, \vartheta, \gamma)$  (see 4.1), and
- $(v_x, v_y, v_z)^l$  GPS- or INS/GPS-derived linear velocity.

The parameters (unknowns) involved in the models are:

- $(X, Y, Z)^l$ ,  $(\omega, \varphi, \kappa)$ ,  $(a_x, a_y, a_z)^c$ ,  $(s_x, s_y, s_z)^l$ , and  $R_c^b(\epsilon_x, \epsilon_y, \epsilon_z)$  (see 4.1), and
- $\delta t$  multi-sensor time synchronization parameter.

And the known constants (instrumental and rotation matrices) are:

- $n$ ,  $R_i^l$ , and  $R_b^{\bar{b}}$  (see 4.1),
- $\Omega_{i\bar{b}}^{\bar{b}}(\omega_x, \omega_y, \omega_z)$  the calibrated (IMU observations corrected with estimated biases, scale factors, etc.) angular velocity matrix from the instrumental  $\bar{b}$  frame with respect to the inertial  $i$  reference frame, and
- $\Omega_{i\bar{l}}^{\bar{b}}(\lambda, \phi, \dot{\lambda}, \dot{\phi}, \omega_e)$  angular velocity matrix which depends on the known position and angular rate of the Earth's rotation<sup>2</sup>.

Depending on the block configuration, the GPS positioning error parameters (commonly referred to as “shift” parameters) can be highly correlated with the multi-sensor time synchronization parameter (see equation 5). One such a case is the usual “one ‘shift’ parameter per strip and constant velocity” configuration. We are currently investigating alternatives that are realistic for everyday routine operations. One possibility would be a single “shift” parameter per block plus the time synchronization parameter. Another possibility would be to fly at a lower speed at the strips ends while taking the, say, first and last images.

## 5.2 Relative Position, Velocity, and Attitude Aerial Control Mathematical Functional Models

The mathematical functional models for the relative position, velocity and attitude aerial control are:

$$\begin{aligned} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}^l - \begin{pmatrix} x_2 + r_{x_2} \\ y_2 + r_{y_2} \\ z_2 + r_{z_2} \end{pmatrix}^l &= \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}^l - \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}^l + \\ &[R_c^l(\omega_1, \varphi_1, \kappa_1) - R_c^l(\omega_2, \varphi_2, \kappa_2)] \cdot \begin{pmatrix} a_x \\ a_y \\ a_z + n \end{pmatrix}^c - \\ &\left[ \begin{pmatrix} v_{x_1} \\ v_{y_1} \\ v_{z_1} \end{pmatrix}^l - \begin{pmatrix} v_{x_2} + r_{v_{x_2}} \\ v_{y_2} + r_{v_{y_2}} \\ v_{z_2} + r_{v_{z_2}} \end{pmatrix}^l \right] \cdot \delta t \end{aligned} \quad (7)$$

$$\begin{aligned} R_c^l(\omega_1, \varphi_1, \kappa_1) \cdot R_i^c(\omega_2, \varphi_2, \kappa_2) &= \\ R_i^l \cdot \left[ R_b^{\bar{b}}(\psi_1, \vartheta_1, \gamma_1) + \dot{R}_b^{\bar{b}}(\psi_1, \vartheta_1, \gamma_1) \cdot \delta t \right] \cdot \\ \left[ R_b^{\bar{b}}(\psi_2 + r_{\psi_2}, \vartheta_2 + r_{\vartheta_2}, \gamma_2 + r_{\gamma_2}) + \dot{R}_b^{\bar{b}}(\psi_2 + r_{\psi_2}, \vartheta_2 + r_{\vartheta_2}, \gamma_2 + r_{\gamma_2}) \cdot \delta t \right]^T \cdot R_i^{\bar{l}} \end{aligned} \quad (8)$$

where

$$\dot{R}_b^{\bar{b}}(\psi_1, \vartheta_1, \gamma_1) = R_b^{\bar{b}}(\psi_1, \vartheta_1, \gamma_1) \cdot \left[ \Omega_{i\bar{b}}^{\bar{b}}(\omega_{x_1}, \omega_{y_1}, \omega_{z_1}) - \Omega_{i\bar{l}}^{\bar{b}}(\lambda_1, \phi_1, \dot{\lambda}_1, \dot{\phi}_1, \omega_e) \right]$$

and

$$\dot{R}_b^{\bar{b}}(\psi_2 + r_{\psi_2}, \vartheta_2 + r_{\vartheta_2}, \gamma_2 + r_{\gamma_2}) = R_b^{\bar{b}}(\psi_2 + r_{\psi_2}, \vartheta_2 + r_{\vartheta_2}, \gamma_2 + r_{\gamma_2}) \cdot \left[ \Omega_{i\bar{b}}^{\bar{b}}(\omega_{x_2}, \omega_{y_2}, \omega_{z_2}) - \Omega_{i\bar{l}}^{\bar{b}}(\lambda_2, \phi_2, \dot{\lambda}_2, \dot{\phi}_2, \omega_e) \right]$$

The involved reference frames are the same as in the previous section (see 5.1).

The observables (measurements) are represented by:

- $(x_2, y_2, z_2)^l$  and  $(\psi_2, \vartheta_2, \gamma_2)$  (see 4.2), and
- $(v_{x_2}, v_{y_2}, v_{z_2})^l$  GPS- or INS/GPS-derived linear velocity at epoch  $t_2$ .

The parameters (unknowns) involved in the models are:

- $(X_1, Y_1, Z_1)^l$ ,  $(\omega_1, \varphi_1, \kappa_1)$ ,  $(X_2, Y_2, Z_2)^l$ , and  $(\omega_2, \varphi_2, \kappa_2)$  (see 4.2),
- $(a_x, a_y, a_z)^c$  (see 4.1), and

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<sup>2</sup>This known matrix can be found in any inertial navigation book, for example [8].

- $\delta t$  (see 5.1).

And the known observational auxiliary values and constants (instrumental and rotation matrices) are:

- $n$  and  $R_T^l$  (see 4.1),
- $(x_1, y_1, z_1)^l$ , and  $(\psi_1, \vartheta_1, \gamma_1)$  (see 4.2),
- $(v_{x_1}, v_{y_1}, v_{z_1})^l$  GPS- or INS/GPS-derived linear velocity at epoch  $t_1$ ,
- $\Omega_{i\bar{b}}^{\bar{b}}(\omega_{x_1}, \omega_{y_1}, \omega_{z_1})$  the calibrated (IMU observations corrected with estimated biases, scale factors, etc.) angular velocity matrix from the instrumental  $\bar{b}$  frame with respect to the inertial  $i$  reference frame at epoch  $t_1$ ,
- $\Omega_{i\bar{l}}^{\bar{b}}(\lambda_1, \phi_1, \dot{\lambda}_1, \dot{\phi}_1, \omega_e)$  angular velocity matrix which depends on the known position at epoch  $t_1$  and angular rate of the Earth's rotation,
- $\Omega_{i\bar{b}}^{\bar{b}}(\omega_{x_2}, \omega_{y_2}, \omega_{z_2})$  the calibrated (IMU observations corrected with estimated biases, scale factors, etc.) angular velocity matrix from the instrumental  $\bar{b}$  frame with respect to the inertial  $i$  reference frame at epoch  $t_2$ , and
- $\Omega_{i\bar{l}}^{\bar{b}}(\lambda_2, \phi_2, \dot{\lambda}_2, \dot{\phi}_2, \omega_e)$  angular velocity matrix which depends on the known position at epoch  $t_2$  and angular rate of the Earth's.

## 6 Conclusions and further research

In this paper we have presented new functional models for a rigorous, simple, comprehensive, reliable and robust use of GPS and INS/GPS kinematic control for the calibration and orientation of multi-sensor data acquisition systems in the context of ISO and DSO.

In some of the new models (equations 4 and 8) certain calibration nuisance parameters like the IMU-to-sensor rotation matrix or the GPS shift parameters are eliminated without loss of correctness. This contributes to simplicity and robustness, as users will no longer concern themselves with them, therefore preventing mistakes. The same models support the isolation of the effects of undetected GPS cycle slips, thus contributing to reliability and robustness. Some models (equations 5, 6, 7, and 8) include a time calibration parameter —also to be interpreted as a temporal-instrumental reference frame transfer parameter— to absorb eventual hardware synchronization errors or to relax hardware design requirements. This adds rigour to the model and improves its reliability and robustness. Note that the estimation of the time calibration parameter requires the knowledge of the sensor linear and angular velocities derived from INS/GPS thus making the use of the INS/GPS information “comprehensive.” The new models are based on already-available INS/GPS output data and, potentially, all sensor systems equipped with an INS/GPS system could benefit from them.

We note that the principles behind the new models —relative control and time calibration with linear and angular velocities— can be applied to derive kinematic control models for other sensors like laser scanners or radars. We note as well, that the models presented can be rewritten for other coordinate systems and parameterizations.

Some of the models presented have already been coded and tested with actual and simulated data. The results obtained so far are consistent and support the above statements. We plan to code the rest of the models and publish the existing and future results.

## 7 Acknowledgements

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