

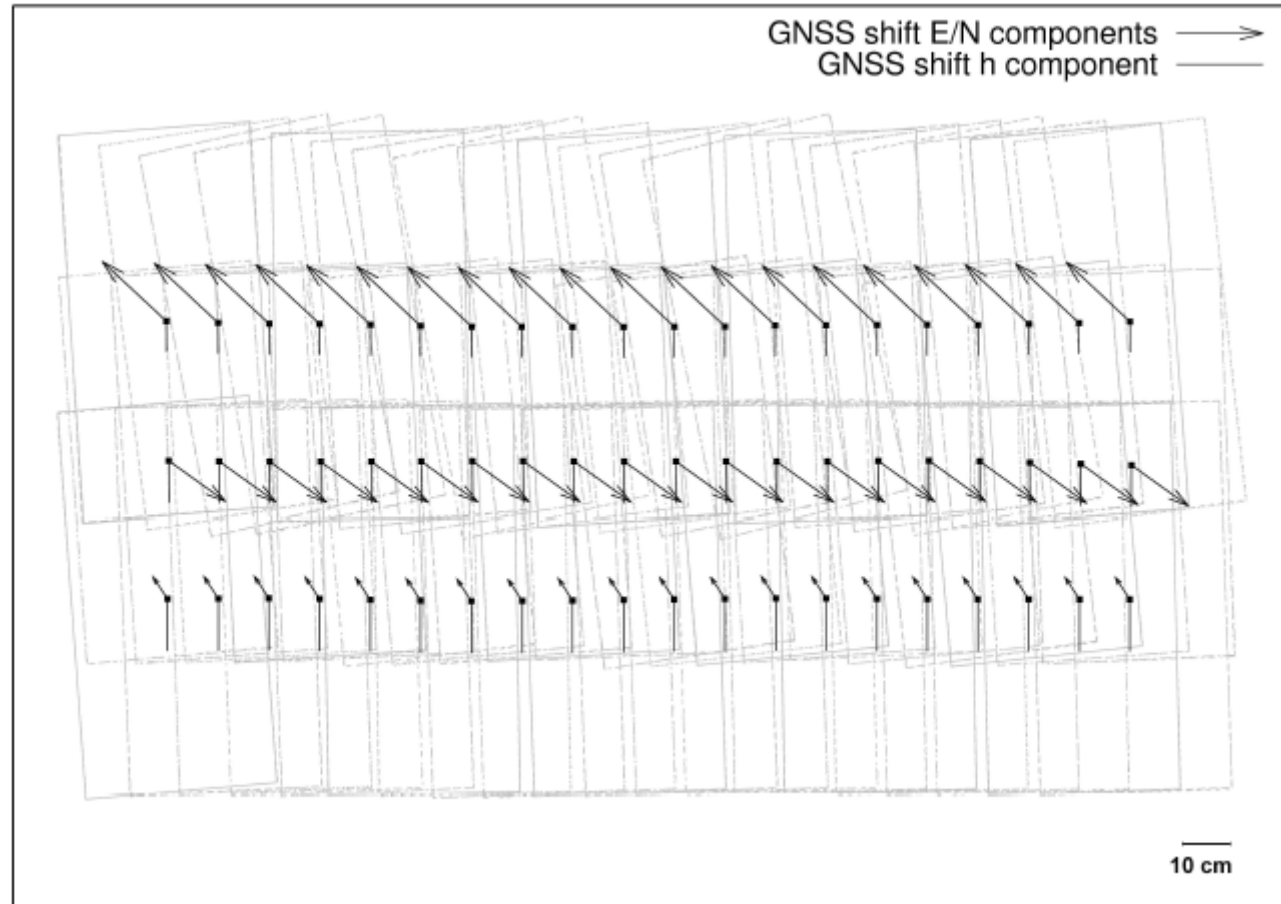


innovating communications

**Pose versus state:  
are sensor position & attitude  
sufficient  
for modern photogrammetry  
and remote sensing?**

*I.Colomina, CTTC*  
*M.Blázquez, GeoNumerics*

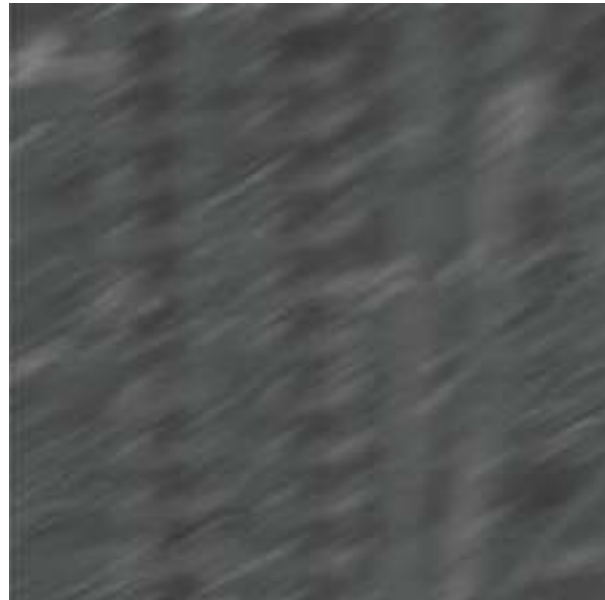
# What is this?



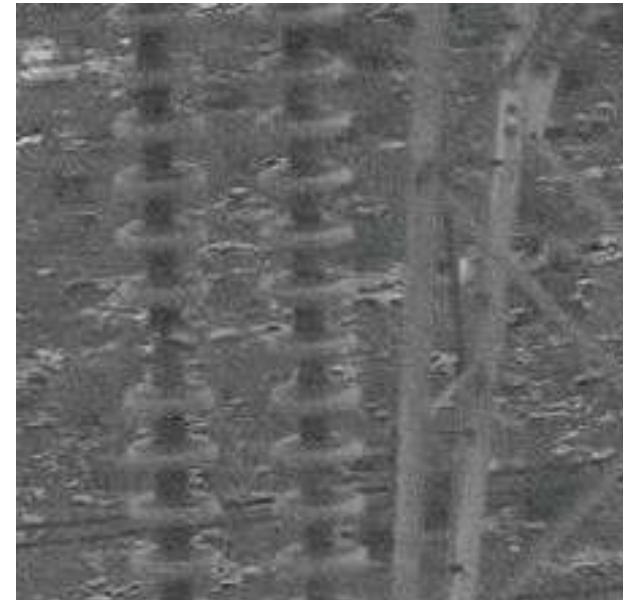
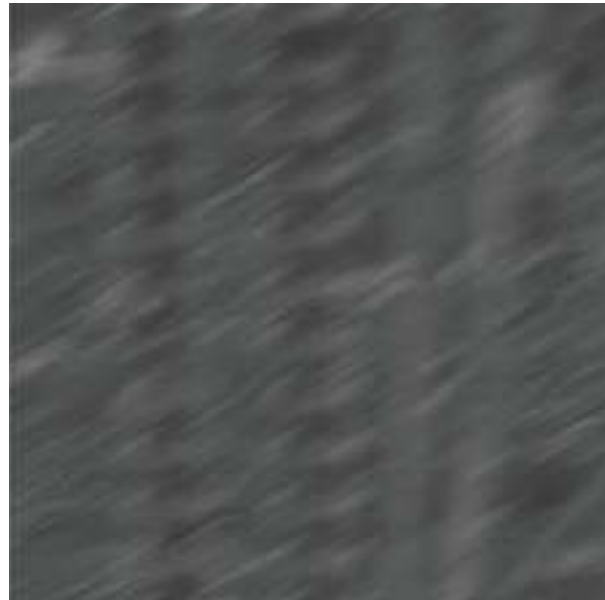


synchronisation error!

# *What is this?*



## *Montion-induced blur.*

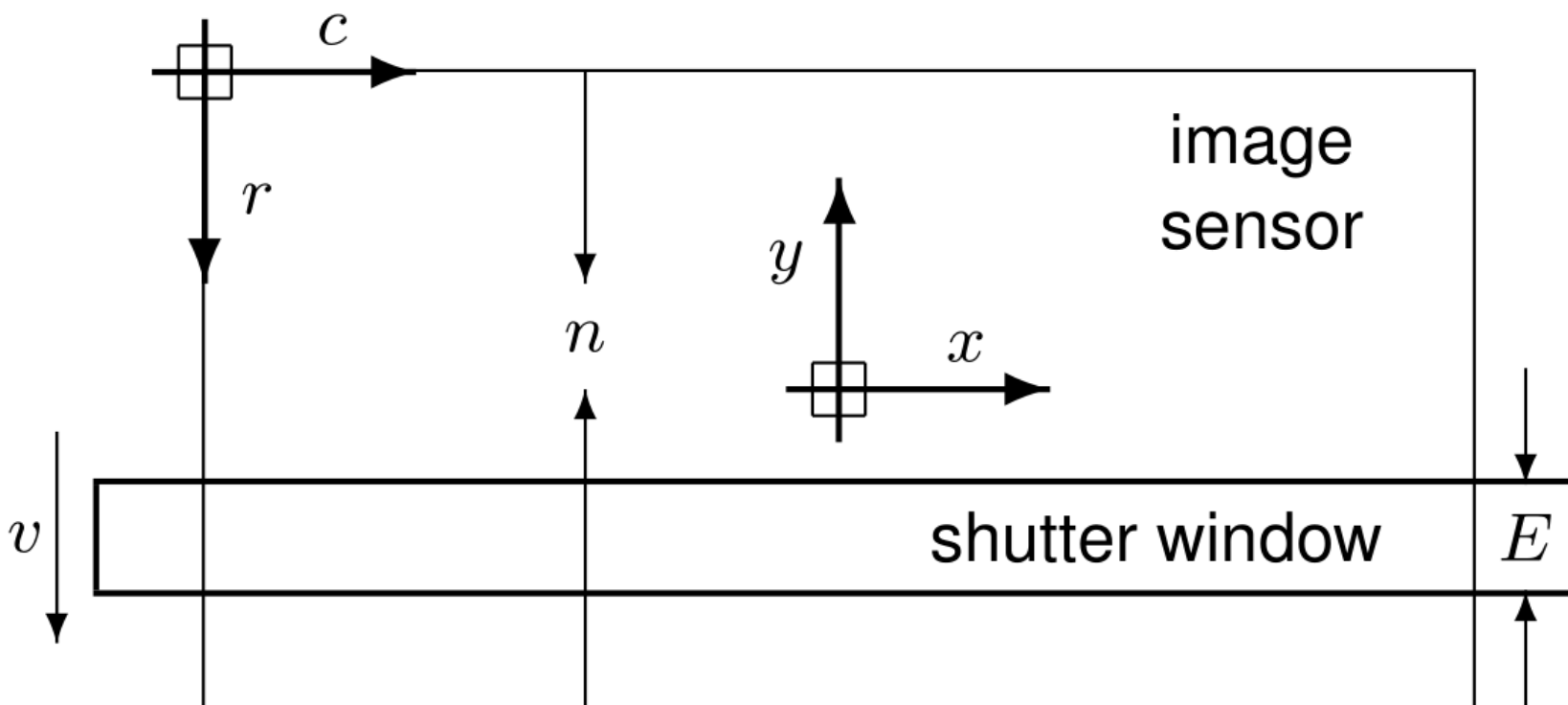


Tong,W-g., Pei, Y-l., Li,B-s., Xu, R-j., 2011. Blurred Image Processing of Aerial Image Based on Improved Wiener Filter and Wavelet Transform. Electric Information and Control Engineering (ICEICE), 2011 International Conference on. Wuhan, China

## *What is this?*



# Focal-plane shutter effect



In a number of problems we were missing the camera's linear & angular velocity.

And therefore we can ask ourselves the question: orientation or orientation++?

1. Sensor state parameters
2. Mathematical structure
3. How to compute them
4. Applications
  - image deblurring
  - sensor-system synchronization
  - focal-plane shutter geometry modeling

## Sensor state parameters

$$s_c^l = (p^l, v^l, \gamma_c^l, \omega_{lc}^c)$$

$$o_c^l = (p^l, \gamma_c^l)$$

$$\dot{p}^l = v^l, \quad \dot{R}(\gamma)_c^l = R(\gamma)_c^l \Omega(\omega)_{lc}^c.$$

## Mathematical structure of $O_c^l$

$$p^l \in \mathbb{R}^3$$

$SO(3)$  special orthogonal group of  $\mathbb{R}^3$

$SE(3)$  special euclidean group of  $\mathbb{R}^3$

$$SE(3) = SO(3) \times \mathbb{R}^3$$

$SE(3)$  is a Lie group under the operator

$$\begin{bmatrix} R(\gamma) & p \\ 0 & 1 \end{bmatrix}$$

# Mathematical structure of $S_c^l$

$$p^l \in \mathbb{R}^3$$

$$v^l \in \mathbb{R}^3$$

$SO(3)$  special orthogonal group of  $\mathbb{R}^3$

$W(3)$  skew symmetric matrices of  $\mathbb{R}^3$

# Mathematical structure of $S_c^l$

$\mathbb{R}^3 \times \mathbb{R}^3 \times SO(3) \times W(3)$  is a group under

$$\begin{bmatrix} R(\gamma) & R(\gamma)\Omega & v \\ 0 & R(\gamma) & p \\ 0 & 0 & 1 \end{bmatrix}$$

## Mathematical structure of $S_c^l$

$$\begin{bmatrix} \varphi'(x) \\ \varphi(x) \\ 1 \end{bmatrix} = \begin{bmatrix} R(\gamma) & R(\gamma)\Omega & v \\ 0 & R(\gamma) & p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ 1 \end{bmatrix} = T \begin{bmatrix} \dot{x} \\ x \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x^l(t) \\ \dot{x}^l(t) \end{bmatrix} = \begin{bmatrix} R(\gamma)_{c(t)}^l x^{c(t)} + p_{c(t)}^l \\ R(\gamma)_{c(t)}^l \Omega_{l c(t)}^{c(t)} x^{c(t)} + R(\gamma)_{c(t)}^l \dot{x}^{c(t)} + \dot{p}_{c(t)}^l \end{bmatrix}$$

## Mathematical structure of $S_c^l$

$$\begin{bmatrix} \varphi'(x) \\ \varphi(x) \\ 1 \end{bmatrix} = \begin{bmatrix} R(\gamma) & R(\gamma)\Omega & v \\ 0 & R(\gamma) & p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ 1 \end{bmatrix} = T \begin{bmatrix} \dot{x} \\ x \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x^l(t) \\ \dot{x}^l(t) \end{bmatrix} = \begin{bmatrix} R(\gamma)_{c(t)}^l x^{c(t)} + p_{c(t)}^l \\ R(\gamma)_{c(t)}^l \Omega_{l c(t)}^{c(t)} x^{c(t)} + \cancel{R(\gamma)_{c(t)}^l \dot{x}^{c(t)}} + \dot{p}_{c(t)}^l \end{bmatrix}$$

## Computation

- from INS/GNSS integration & system calibration

$$\Omega(\omega)_{lc}^c = R_b^c \Omega(\omega)_{lb}^b R_c^b$$

plus two strategies:

- extend the INS differential eq. of motion;
- or correct angular rates with calibration states

- from orientation parameters in time-tagged image sequences

$$(t_k - t_{k-1}) \Omega_{k-1,k}^k = \log R_{k-1}^k$$

## Computation

- from orientation parameters in time-tagged image sequences

$$(t_k - t_{k-1})\Omega_{k-1,k}^k = \log R_{k-1}^k$$

$$\Omega(\omega)_{lk}^k = R_{k-1}^k \Omega_{l,k-1}^{k-1} R_k^{k-1} + \Omega_{k-1,k}^k$$

## *APPS: motion deblurring*

- Blurred and latent images. Blur kernel.

$$B + v = L \otimes k$$

- Deconvolution: integral equation
  - blind deconvolution:  $L, k$  unknown
  - non-blind deconvolution:  $L$  unknown
- Blur kernel

$$\int \int k(x, y) dx dy = 1$$

$$k(x, y) = P(x, y, e) = \{(x + \delta_x(\tau), y + \delta_y(\tau))\}_{\tau \in [-e/2, e/2]}$$

## *APPS: motion deblurring*

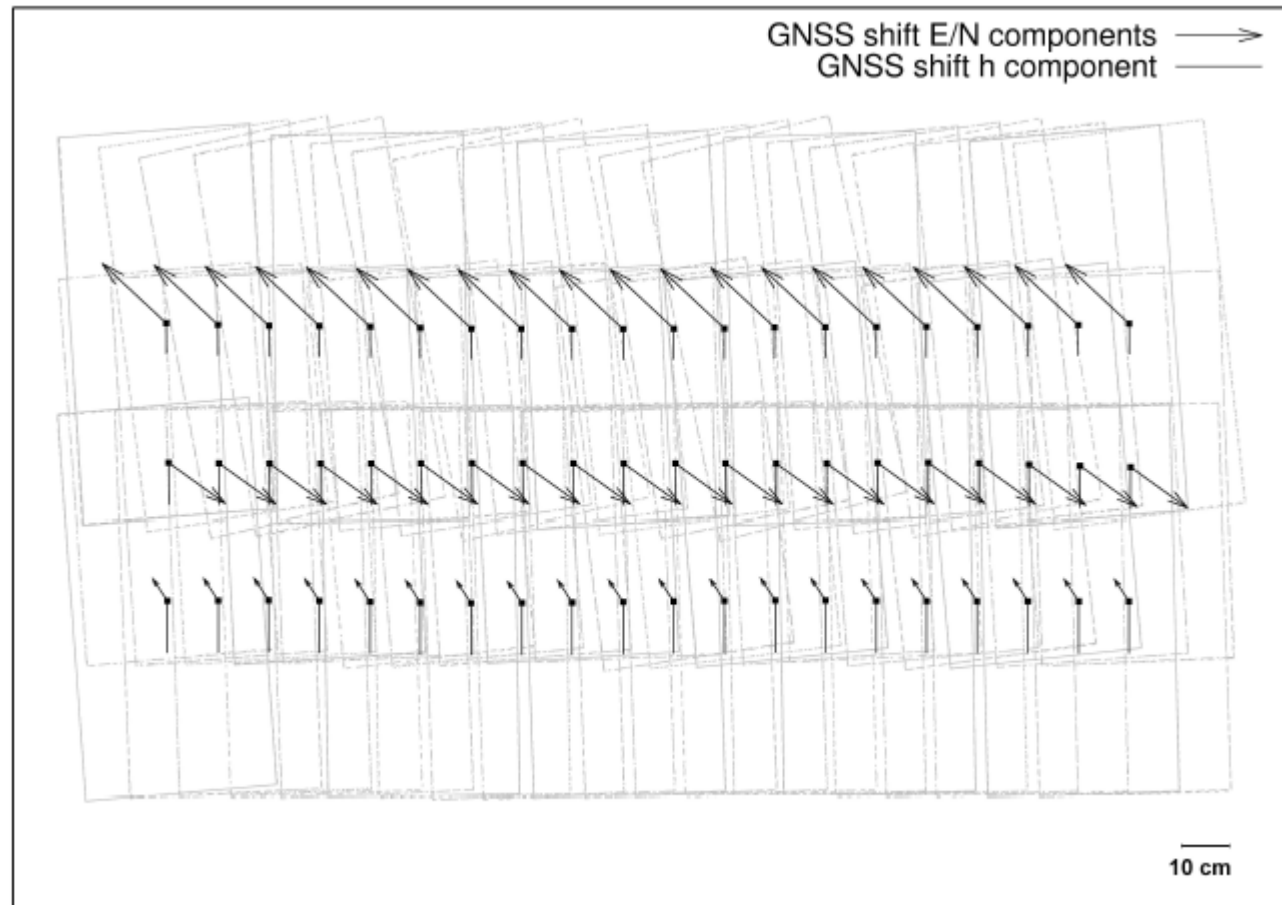
- In general,  $P(x,y,e)$  is a curve

$$F(s_c, x^l, p^l(t + \tau), R_l^c(\gamma(t + \tau)))$$

- It can be approximated by a straight-line

$$\begin{bmatrix} x \\ y \end{bmatrix} + \frac{\partial F}{\partial (s_c, x^l, p^l(t), R_l^c(t))} \cdot \frac{\partial (s_c, x^l, p^l(t), R_l^c(t))}{\partial \tau} \cdot \tau$$

# *APPS: synchronisation*



# synchronisation

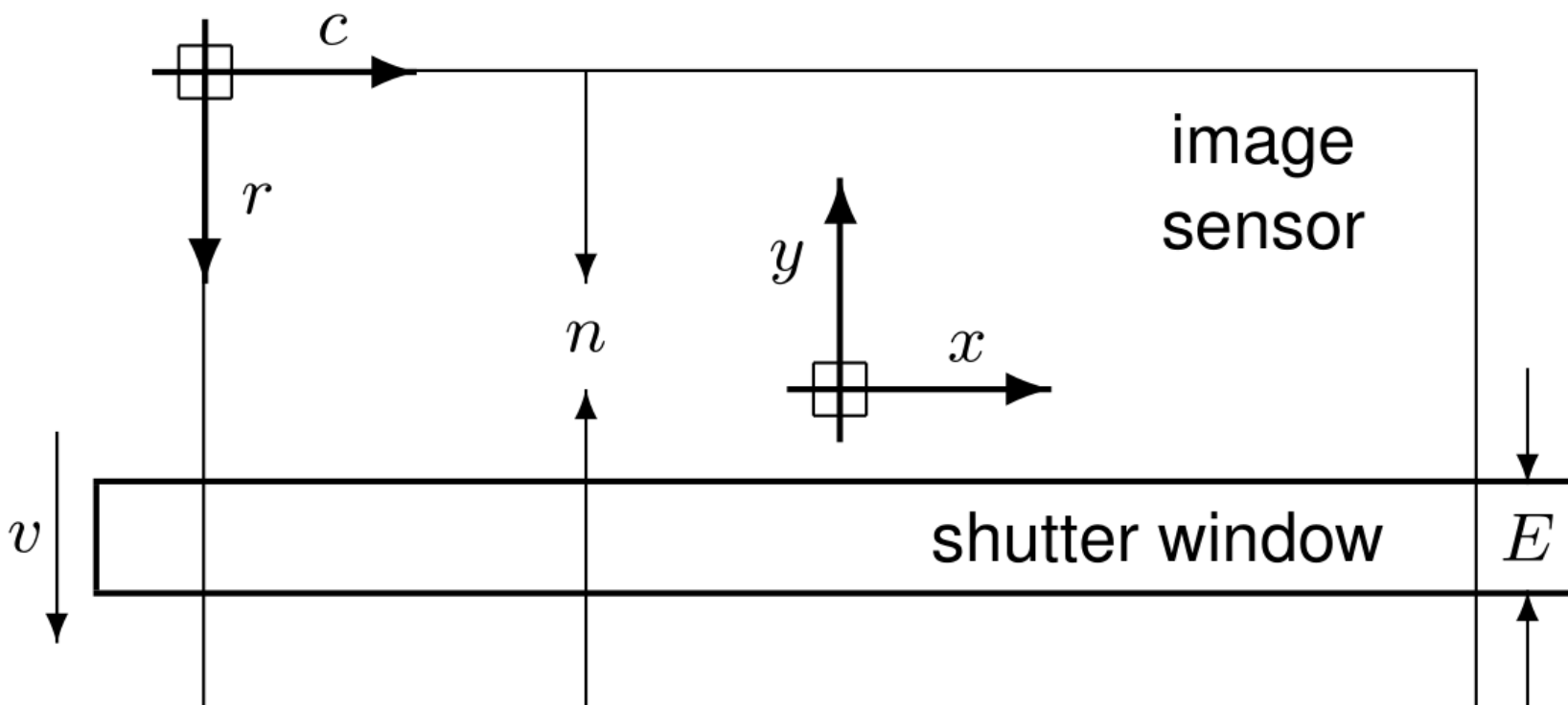
$$\tilde{\mathbf{x}}^l + \tilde{\mathbf{v}}_x^l =$$

$$\tilde{\mathbf{X}}^l + \mathbf{R}_c^l(\Gamma) \cdot (\tilde{\mathbf{A}}^c + \tilde{\mathbf{N}}^c) + \tilde{\mathbf{S}}^l - (\tilde{\mathbf{v}}^l + \tilde{\mathbf{v}}_v^l) \cdot \Delta t,$$

$$\mathbf{R}_c^l(\Gamma) =$$

$$\mathbf{R}_{l'}^l \cdot [\mathbf{R}_{b'}^{l'}(\chi + \tilde{\mathbf{v}}_\chi) + \dot{\mathbf{R}}_{b'}^{l'}(\chi + \tilde{\mathbf{v}}_\chi) \cdot \Delta t] \cdot \mathbf{R}_b^{b'} \cdot \mathbf{R}_c^b(\Upsilon)$$

# *APPS: focal-plane shutter*



## Focal-plane shutter model

$$x^l = p^l + \mu \cdot R_c^l \begin{bmatrix} x \\ y \\ -f \end{bmatrix}^c$$

$$x^l = p^l(x, y) + \mu \cdot R_c^l(x, y) \begin{bmatrix} x \\ y \\ -f \end{bmatrix}^c$$

$$p^l(x, y) = p^l + \Delta t(y) \cdot v^l,$$

$$R_c^l(x, y) = R_c^l \cdot (I + \Delta t(y) \cdot R(\omega)_{lc}^c).$$

# CONCLUSIONS

- $(p, v, \gamma, \omega)$  extend  $(p, \gamma)$
- Sensor state parameters (extension of sensor orientation) have a similar [group] mathematical structure to orientation
- They can be easily estimated from INS/GNSS
- Also from image sequences (?) ,,,
- We presented ways to use them in
  - Motion-based image deblurring
  - Sensor-system synchronisation
  - Modeling focal-plane shutter geometry
- We still need to understand better the mathematical structure of  $(p, v, \gamma, \omega)$