Singular Value Decomposition-based Robust Cubature Kalman Filtering for an Integrated GPS/SINS Navigation System

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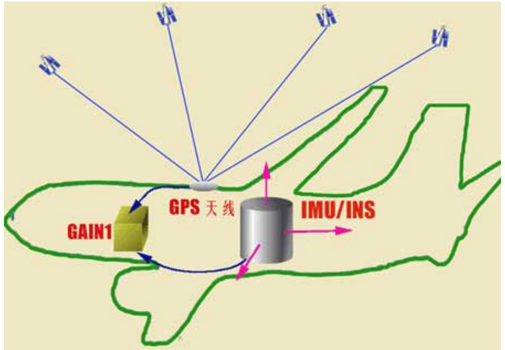
1 Introduction

GPS/SINS navigation system

- Aerial Photogrammetry and RS
- Mobile Mapping System
- Precision Agriculture
- Intelligent Vehicles



1 Introduction

- Standard Kalman filtering (KF)
  - Linear system (✔)
  - Non-linear system (✗)
  - Low-cost SINS error model (✗)
  - Alignment with large misalignment angle (✗)

- Non-linear Kalman filtering
  - Extended KF (only small nonlinearity model)
  - Unscented KF (need to set parameters correctly)
  - Particle filtering (heavy computation burden)
  - Cubature Kalman filtering (CKF, adopted)

1 Introduction

- Standard Kalman filtering

  All errors being Gaussian noise (✓)

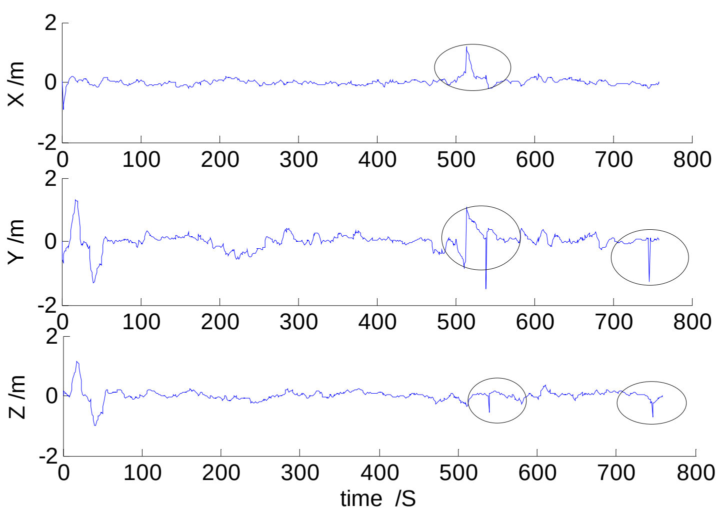
  Known dynamic system (process mode)

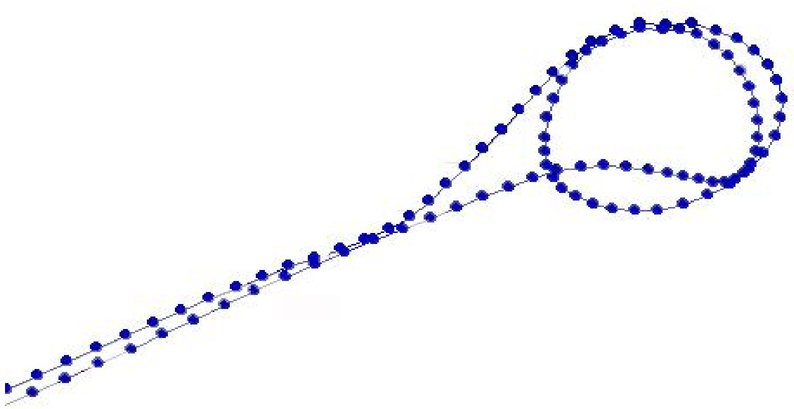
  and measurement models (sensor model) (✓)

  Outlier data (✗)

  Model uncertainty (✗)

- Robust Kalman filtering (✓, adopted)





2 Robust Cubature Kalman Filtering

Standard CKF

$$\boldsymbol{K}_k = \boldsymbol{P}_{xz,k} / \boldsymbol{P}_{zz,k}$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k/k-1} - \boldsymbol{K}_k \boldsymbol{P}_{zz,k} \boldsymbol{K}_k^T$$

$$\boldsymbol{J} = \frac{\sum_{k=1}^{N} \left\| \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \right\|^2}{\left\| \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 \right\|^2_{\boldsymbol{P}_0^{-1}} + \sum_{k=1}^{N} \left(\left\| \boldsymbol{w}_k \right\|^2_{\boldsymbol{Q}_k^{-1}} + \left\| \boldsymbol{v}_k \right\|^2_{\boldsymbol{R}_k^{-1}} \right)}$$

Robust CKF

$$\boldsymbol{P}_k = \boldsymbol{P}_{k/k-1} - \begin{bmatrix} \boldsymbol{P}_{xz,k} & \boldsymbol{P}_{k/k-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_{zz,k} - \boldsymbol{R}_k + \boldsymbol{I} & \boldsymbol{P}_{xz,k}^T \\ \boldsymbol{P}_{xz,k} & \boldsymbol{P}_{k/k-1} - \gamma^2 \boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{P}_{xz,k}^T \\ \boldsymbol{P}_{k/k-1}^T \end{bmatrix}$$

Significant parameter
Distinguished with other filtering algorithms

3 Case study 1

Table 1: IMU technical specifications

	SPAN-CPT	SPAN-LCI
Gyro Rate Bias	20 deg/hr	<1.0 deg/hr
Gyro Scale Factor	1500ppm	500ppm
Angle RW	0.067 deg/rt-hr	<0.15 deg/rt-hr
Acc. Bias	50mg	<1.0 mg
Acc. Scale Factor	4000ppm	<1000ppm
Velocity RW	55ug/rt-Hz	——

SPAN-CPT

3 Case study 1

Fig. 1: The vehicle trajectory

Tab. 2: Position errors of different filters

	RMS of Position Error (m)		
	X	Y	Z
CKF	0.129	0.229	0.116

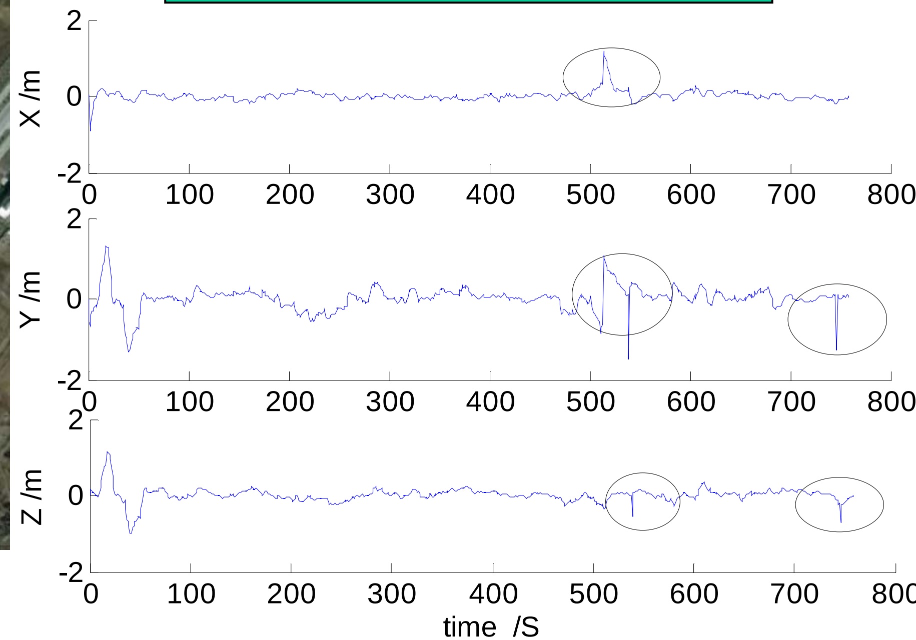


Fig. 2: The position error of CKF

3 Case study 1

Table 2: Position errors ($\gamma = 2$)

	RMS of Position Error (m)		
	X	Y	Z
CKF	0.129	0.229	0.116
RCKF	0.090	0.175	0.107
SVD- RCKF	0.090	0.175	0.107

The robust filter is effective.

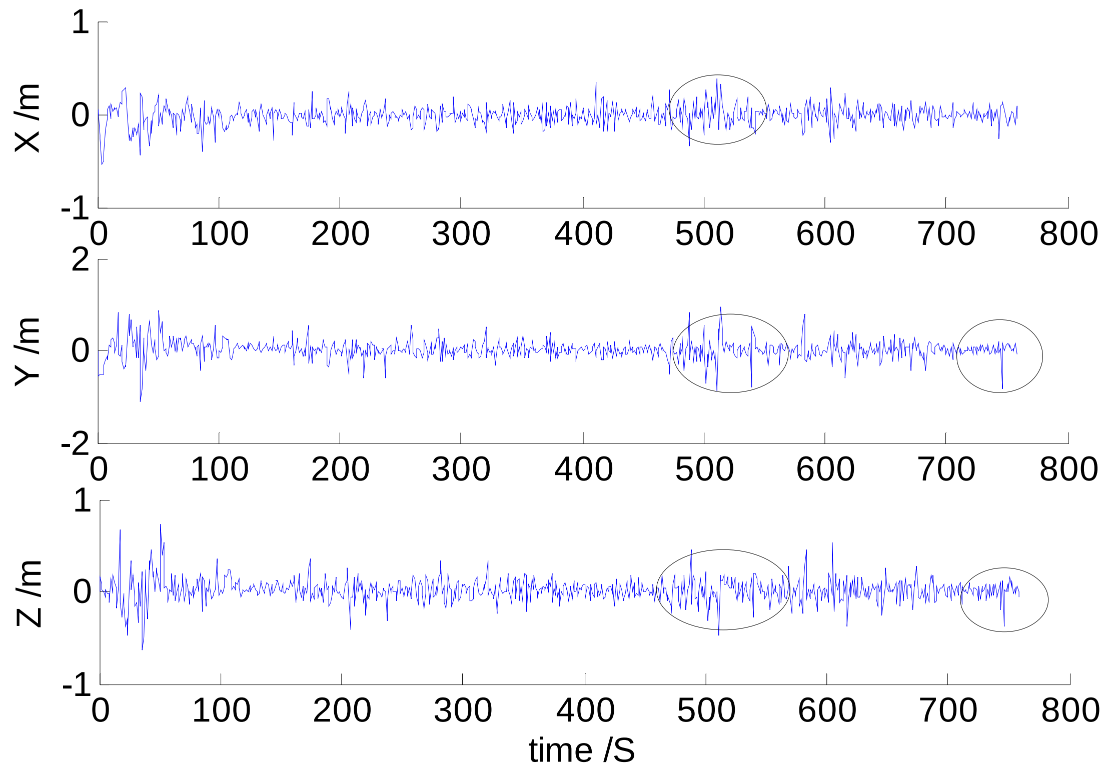


Fig. 3: The position error of RCKF (γ = 2)

3 Case study 1

Tab. 3: The position errors of different strict parameter

Restrict parameter		0.5	0.7	1.0	1.4	1.5	2.0	2.5	25	250	2500
RCKF (Position Error in m)	X	---	---	---	---	0.087	0.091	0.092	0.093	0.093	0.093
	Y	---	---	---	---	0.168	0.176	0.178	0.181	0.181	0.181
	Z	---	---	---	---	0.105	0.108	0.108	0.109	0.109	0.109

For the RCKF, the larger value the parameter is, the less the robustness of the filter.

RCKF can get the jarless robustness performance if the different design parameters are within limits.

3 Case study 1

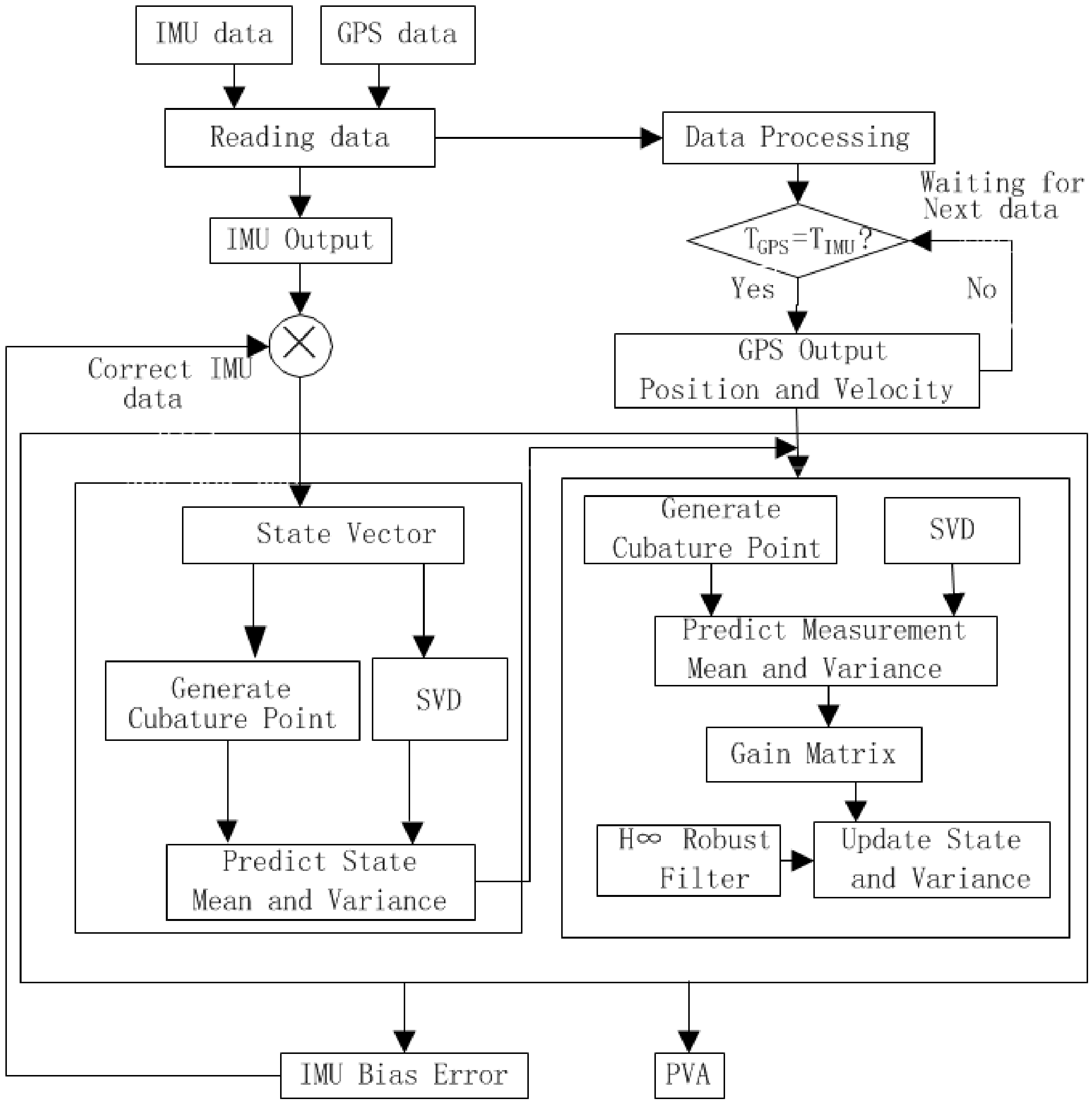
Tab. 3: The position errors of different strict parameter

Restrict parameter		0.5	0.7	1.0	1.4	1.5	2.0	2.5	25	250	2500
RCKF (Position Error in m)	X	---	---	---	---	0.087	0.091	0.092	0.093	0.093	0.093
	Y	---	---	---	---	0.168	0.176	0.178	0.181	0.181	0.181
	Z	---	---	---	---	0.105	0.108	0.108	0.109	0.109	0.109

Why? $P_{k/k-1}$ and P_k lose their positive definiteness

How? Singular Value Decomposition

4 SVD-RCKF



Flow Chat of SVD-RCKF

4 SVD-RCKF

Tab. 2: Position errors of different filters (γ =2)

	RMS of Position Error (m)		
	X	Y	Z
CKF	0.129	0.229	0.116
RCKF	0.090	0.175	0.107
SVD- RCKF	0.090	0.175	0.107

Tab. 3: The position errors of different strict parameter

Restrict parameter		0.5	0.7	1.0	1.4	1.5	2.0	2.5	25	250	2500
SVD- RCKF (Position Error in m)	X	0.850	0.554	0.04	0.088	0.087	0.091	0.092	0.093	0.093	0.093
	Y	1.013	0.394	0.069	0.167	0.168	0.176	0.178	0.182	0.182	0.182
	Z	0.851	0.379	0.053	0.105	0.105	0.108	0.109	0.109	0.109	0.109
RCKF (Position Error in m)	X	---	---	---	---	0.087	0.090	0.092	0.093	0.093	0.093
	Y	---	---	---	---	0.168	0.175	0.178	0.181	0.181	0.181
	Z	---	---	---	---	0.105	0.107	0.108	0.109	0.109	0.109

4 SVD-RCKF

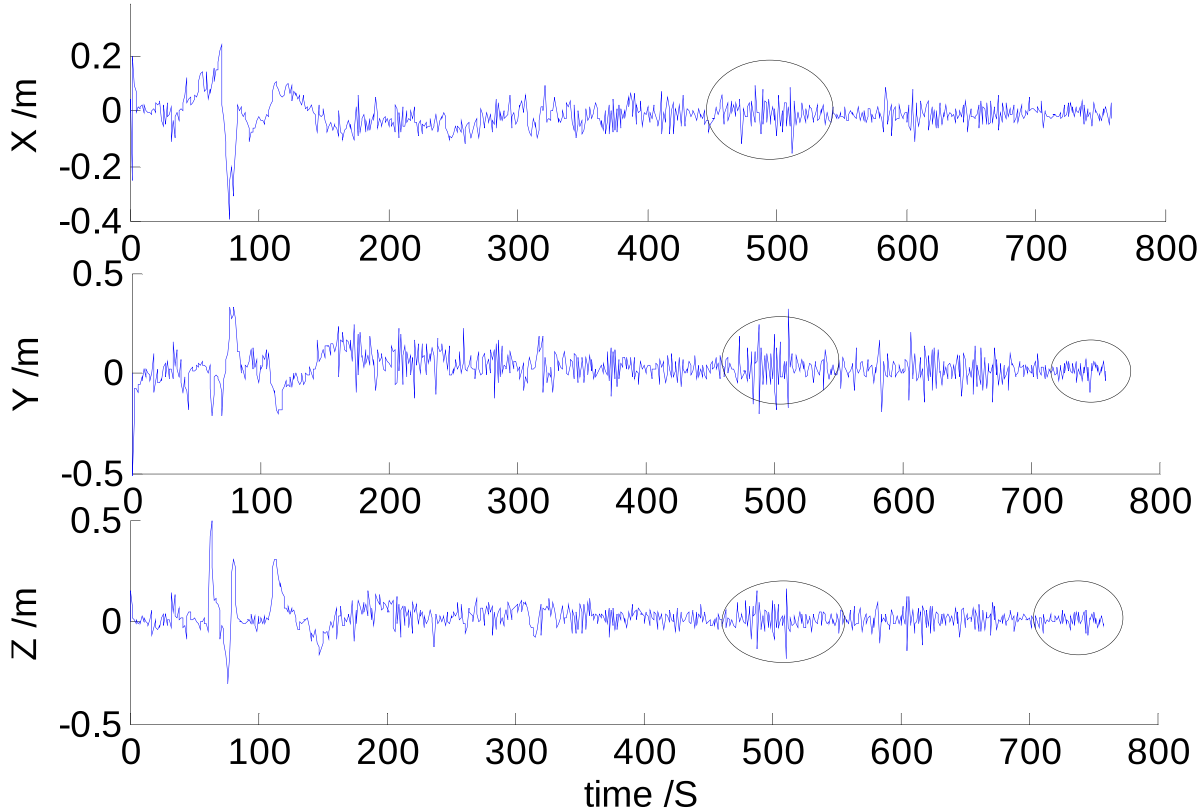


Fig. 4: The position error of SVD-RCKF (γ =1)

The robust cubature Kalman filter based on SVD can maintain the system stability and get more robustness on the wider conditions for the design parameters.

5 Case study 2

SPAN-LCI

Tab.1: IMU technical specifications

	SPAN-CPT	SPAN-LCI
Gyro Rate Bias	20 deg/hr	<1.0 deg/hr
Gyro Scale Factor	1500ppm	500ppm
Angle RW	0.067 deg/rt-hr	<0.15 deg/rt-hr
Acc. Bias	50mg	<1.0 mg
Acc. Scale Factor	4000ppm	<1000ppm
Velocity RW	55ug/rt-Hz	——

Fig. 6. The testing van

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Fig. 5: The vehicle trajectory

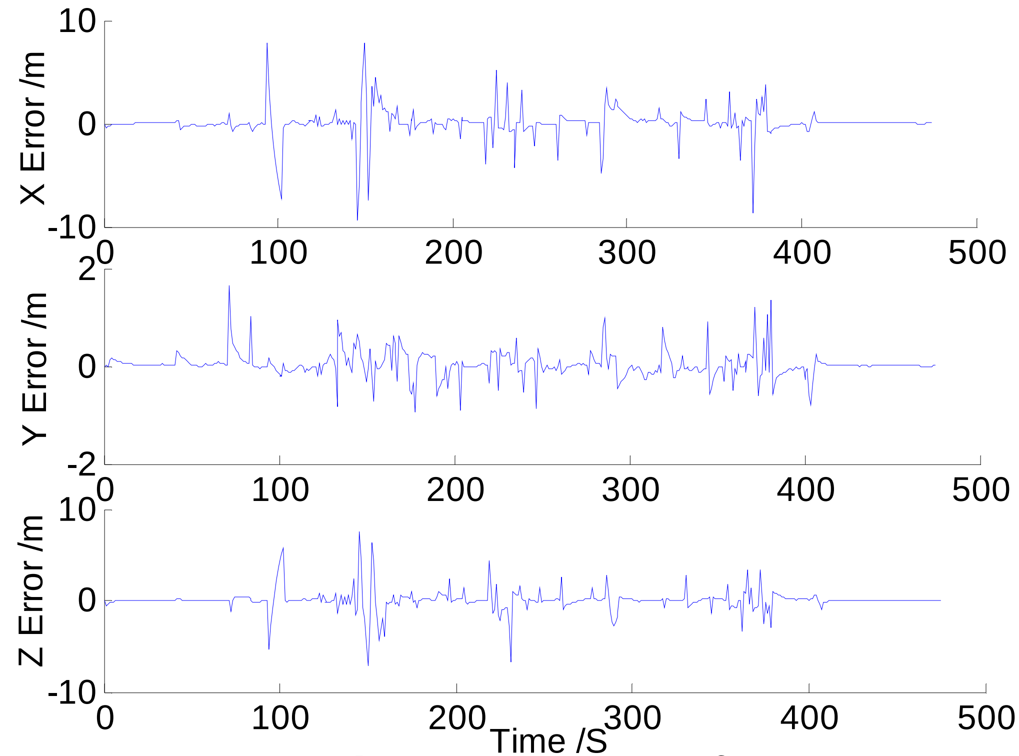

Fig. 6: The position error of CKF

Tab. 4: Position errors of different filters

	RMS of Position Error(m)		
	X	Y	Z
CKF	1.430	0.264	1.185

5 Case study 2

Tab. 4: Position errors of different filters
(gamma = 3)

	RMS of Position Error(m)		
	X	Y	Z
CKF	1.430	0.264	1.185
RCKF	0.066	0.016	0.061
SVD- RCKF	0.066	0.016	0.061

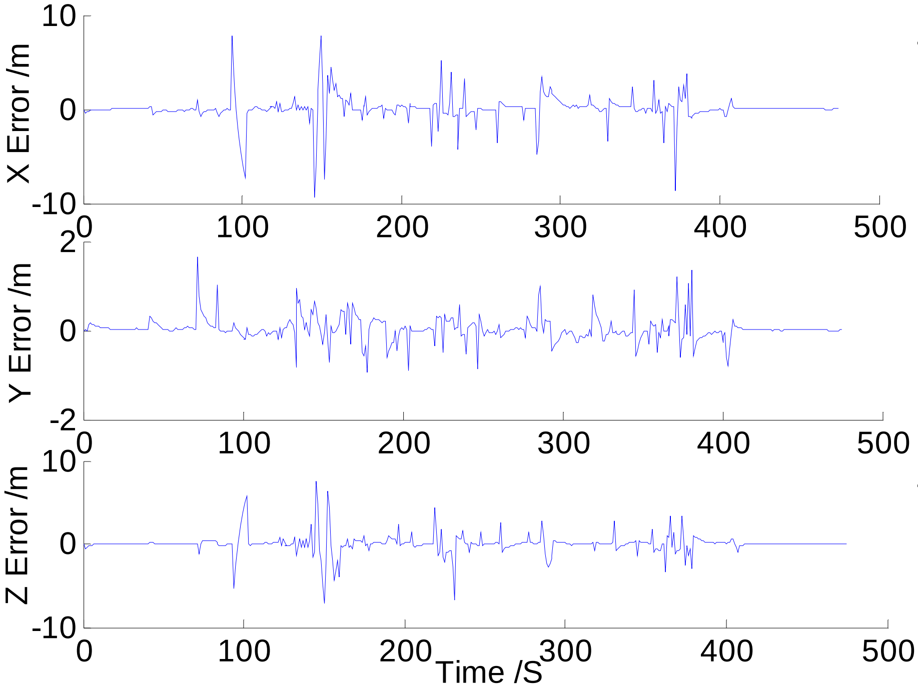

Fig. 6: The position error of CKF

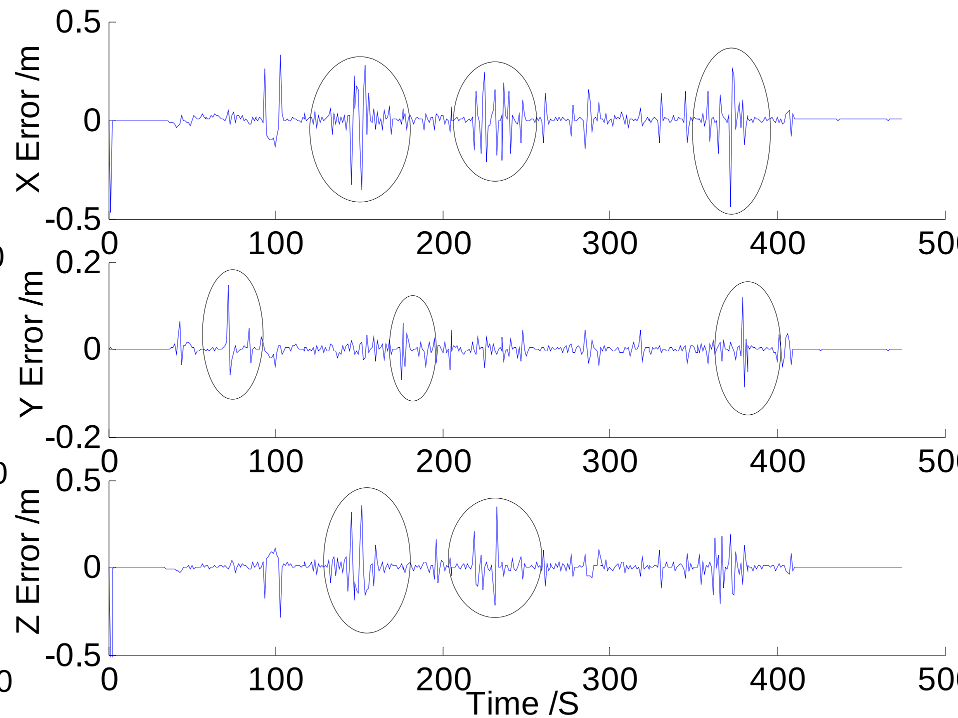

Fig. 7: The position error of RCKF (gamma = 3)

5 Case study 2

Tab. 5: The position errors of different strict parameter

Restrict parameter		0.8	0.866	1	1.414	1.732	2	3	4	5	50	500	5000
SVD- RCKF (Position Error /m)	X	1.576	0.375	0.112	0.046	0.055	0.059	0.066	0.068	0.069	0.071	0.071	0.071
	Y	1.632	0.362	0.118	0.011	0.016	0.018	0.016	0.016	0.016	0.016	0.016	0.016
	Z	2.630	0.333	0.277	0.045	0.052	0.056	0.061	0.063	0.063	0.065	0.065	0.065
RCKF (Position Error /m)	X	---	---	---	---	---	---	0.066	0.068	0.069	0.071	0.071	0.071
	Y	---	---	---	---	---	---	0.016	0.016	0.016	0.017	0.017	0.017
	Z	---	---	---	---	---	---	0.061	0.063	0.063	0.065	0.065	0.065

5 Case study 2

γ

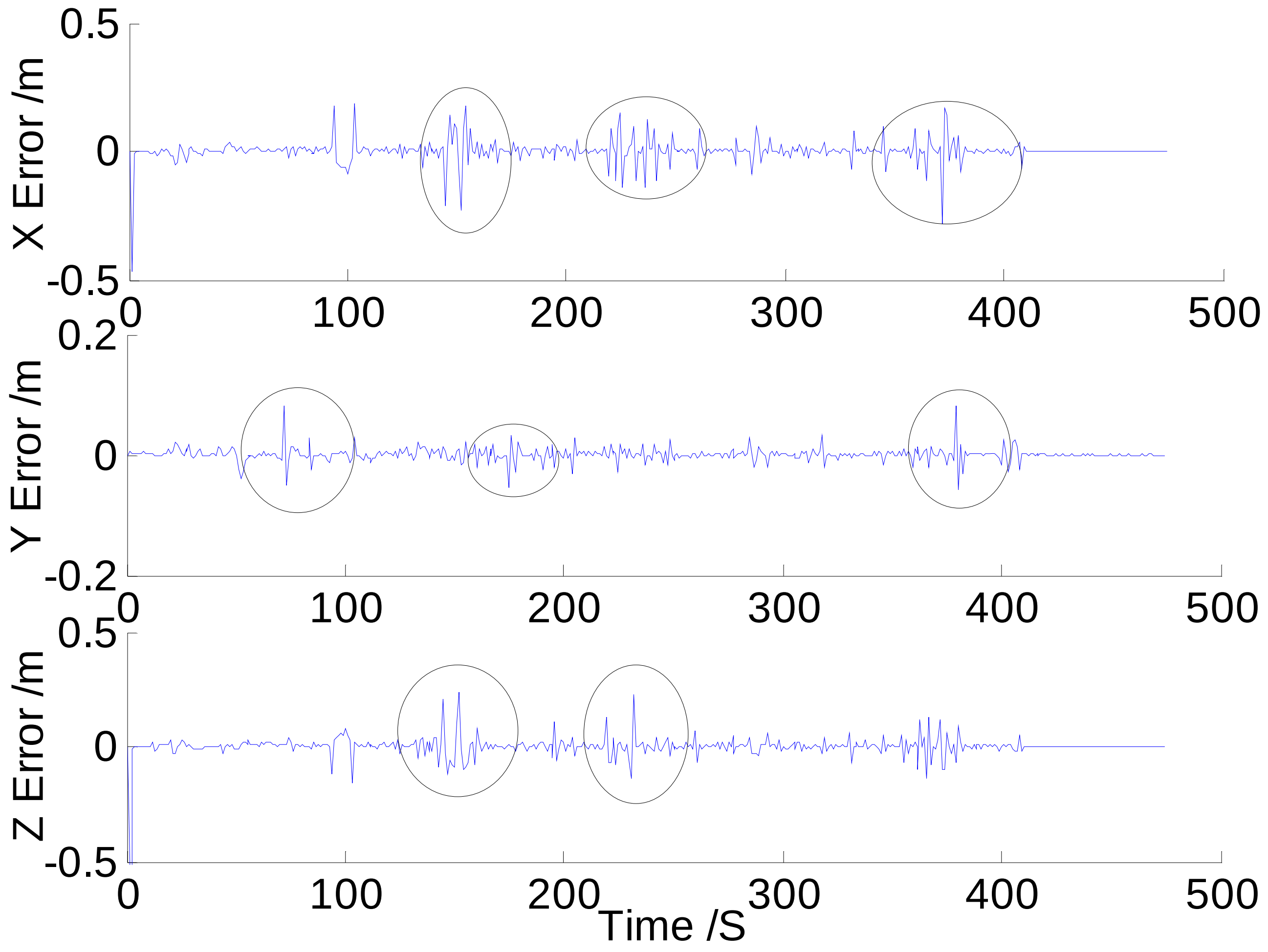

Fig. 7: The position error of RCKF (gamma = 1.414)

6 Conclusions

- The robust cubature Kalman filtering based on the H_∞ filter is very effective for detecting outlier data in GPS/SINS integration system.

- It has been found that the smaller restrict parameter can improve the overall performance of RCKF.

6 Conclusions

- However, it is also apparent that RCKF is easy to diverge if the parameter is much small.
- The robust cubature Kalman filtering based on SVD can keep maintain the system stability and get more robustness on the wider conditions for the design parameters.
- How to set the optimal parameter is the future work.

Thank You!

Any Questions?

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